Can Consumer Complaints Reduce Product Reliability?

Joaquín Coleff *

Abstract

In their dealings with retailers and suppliers, regulations and warranties ensure that consumers can seek a repair, a replacement or a refund if the good they have purchased is faulty. The evidence, however, indicates that few consumers pursue any form of compensation, suggesting that, for some consumers, transaction costs are high and providing a rationale for the role that consumers’ associations play. In this article, we analyze the monopolist’s pricing and product reliability problem when consumers are entitled to product replacement but have heterogeneous cost of exercising this right, and we assess the implications of a decrease in consumers’ claiming costs. In this environment, the firm increases product reliability to increase consumers’ willingness to pay and to reduce the expected number of complaints. As consumers claiming cost decreases, the number of complaints and the consumer surplus increase. If this effect is sufficiently high, the firm can partially appropriate the surplus that consumers derive from replacing defected units by increasing its price. However, as consumers claiming cost decreases, the effect of product reliability on consumer willingness to pay diminishes, and, thus, product reliability may decrease. Surprisingly, consumers complaints may reduce product reliability. Our results, that are robust to different specifications, help to explain why some firms help and some others discourage consumers from complaining. Our simple model also shows that firms have incentives to overprovide product reliability. Finally, our results have several suggestions and policy implications.

Keywords: product reliability, consumers’ complaints, liability cost, warranty.
JEL Classification: K42,D71, D42, D21, L12.

*Universidad del Rosario, jcoleff@gmail.com. I am indebted to my advisor Susanna Esteban for her support and helpful guide. I also thank Marc Möller, Carlos Ponce, Marco Celentani, Antonio Cabrales, Daniel García-González, Walter Cont, Juan José Gauza, Marc Vorsatz, Gerard Llobet, Andrew Daughety, Jennifer Reinganum, and Dolores de la Mata for helpful comments. Also, I would like to thank participants in seminars at University Carlos III de Madrid, Universidad del Rosario, Universidad de los Andes, Universidad Alberto Hurtado, Jamboree 2010 at Toulouse School of Economics (ENTER), LACEA meeting 2011, and Simposio XXXV of SAEe 2011. All remaining errors are my own.
1 Introduction

In their dealings with retailers and suppliers, regulations and warranties ensure that consumers can seek a repair, a replacement or a refund if the good they have purchased is faulty (Mann and Wissink, 1990). The evidence, however, indicates that few consumers pursue any form of compensation (Andreasen and Best, 1977; Huppertz, 2007), suggesting that, for some consumers, claiming costs are high (Huppertz, 2007; Hirschman, 1970) and providing a rationale for the role that consumers’ associations play in helping consumers channel their complaints. In this article, we analyze the monopolist’s pricing and product reliability problem when consumers are entitled to product replacement and we assess the implications of a decrease in consumers’ claiming costs due to, for instance, the appearance of internet and/or consumers’ associations. Our results suggest that product reliability may be decreased after a reduction of consumers claiming cost.

There are several facts that facilitates consumers to engage in claims when receiving faulty units. First, consumer’s rights are constantly improving.\(^1\) Second, there are several mechanisms that facilitate complaints: class action lawsuits, small courts, public agencies (e.g., NHTSA in US) and consumer associations (claiming on consumer’s behalf). Third, information and communication technologies (ICT) allow for private enterprises: “getsatisfaction.com” in US provides a service to handle complaints and mediate between consumers and firms. Another interesting site, miqueja.com in Spain, publishes consumer’s complaints and firms’ responses, grading firms according to the number of complaints received and how they respond to them. Finally, it is also getting important for firms to handle correctly consumer’s complaints: more than 40,000 firms use “getsatisfaction.com” in US.\(^2\)

To study the effect of a reduction in the consumers’ claiming cost on the firm’s decisions, we construct a model with a monopolist choosing the price and the reliability of the product it manufactures. The product’s reliability is defined by the probability that it is not defective. Replacing a faulty product, and providing a more reliable one are costly actions for the firm. Consumers derive a high utility from consuming a product that does not break down, and a low utility otherwise. If a product is faulty, the consumer chooses whether to seek a replacement, incurring a cost. Consumers differ in their claiming cost. Within this set-up, we propose a comparative static analysis to understand the impact of a decrease in consumers’ claiming cost (e.g., moving downward the distribution of consumers claiming cost) on reliability, price and firm’s profits. We show that the monopolist may optimally respond to a decrease in transaction costs by decreasing the product’s reliability.

In this environment it is efficient that all consumers are served. However, as consumers have heterogeneous claiming cost, the firm cannot fully appropriate the benefits that generates to consumers.\(^3\) As a

\(^{1}\)In 1962, Kennedy’s Administration in US introduced the first consumer’s rights: Safety, Choose, Be Informed, and Be Heard. Another rights were included in 1985: Satisfaction of Basic Needs, Redress, Consumer Education, and Healthy Environment. More rights are constantly improving consumers conditions. These rights fastly spread to most countries.

\(^{2}\)Among these firms you find Procter and Gamble (Pampers), Microsoft, Panasonic and Amazon.

\(^{3}\)In our simple environment, there are two benefits, one of selling the product and another of replacing defected units. Heterogeneity is placed in the benefit of replacing defected units. The heterogeneity in consumers claiming cost can be interpreted as the firm having asymmetric information about the consumer’s claiming cost.
consequence, the firm may have incentives to choose product price and reliability such that it excludes some consumers from buying. We call this case a high-pricing strategy, otherwise we say that the firm follows a low-pricing strategy. The firm would choose a high-pricing strategy when consumers claiming cost is sufficiently low, i.e., when the benefit generated to buyers is highest. In this case, the additional surplus appropriated per unit sold compensate the lower profit for excluding some consumers.\footnote{Dispersion of consumers’ willingness to pay is negatively related with consumers’ claiming cost, and in line with Johnson and Myatt (2006), “firm’s profits are a U-shaped function of dispersion [of consumers’ willingness to pay]. High dispersion is complemented by niche production and low dispersion is complemented by mass-market supply.” (brackets added)}

The firm’s strategy that defines the product price and reliability depends on consumers claiming cost. In any case, the incentives to provide costly reliability comes from increasing the willingness to pay of the marginal consumer and from reducing the cost associated with replacements. In counterpart, it is more expensive to manufacture a more reliable product.

The main result of the article states that product reliability may decrease when consumers claiming cost decreases. If consumers claiming cost is low, the firm will follow a high-pricing strategy charging a high price that reduces demand. Increasing product reliability decreases the expected number of replacements, and increases both consumers’ willingness to pay, allowing to charge a higher price, and demand. Note that consumers’ willingness to pay internalizes the expected future cost associated to replacements. Replacements are less likely if the product is more reliable and they are less costly (for consumers) if complaints are cheaper. Consequently, the effect of product reliability on willingness to pay and on demand diminish as consumers claiming cost decreases. Then, product reliability decreases as consumers claiming cost decreases. Note also that demand, price and profits increase when consumers claiming cost decreases.

If consumers claiming cost is high, the firm will follow a low-pricing strategy. In this case, demand is high and the price is limited by the willingness to pay of the marginal consumer who does not find rewardable replacing defected units. However, there are some consumers, defined as claimants, that always replace defected units. Increasing product reliability reduces the expected number of replacements and increases the willingness to pay of the marginal consumer. The number of claimants, and thus, the incentives to provide product reliability increase as consumers claiming cost decreases. In this case, a marginal reduction in consumers claiming cost increases the number of claimants and product reliability, and reduces firm’s profit (which coincide with common wisdom). However, the more the claimants the higher the surplus consumers make by replacing defected units. If there is a non-locally reduction in consumers claiming cost, the firm may change from low to high-pricing strategy. This change modifies the incentives to provide product reliability as described in the previous paragraph, and, eventually, product reliability can decrease as consumers claiming cost decreases.

This analysis reveals that not only product failure rate but also firm’s profits have a U-shaped relation with consumers claiming cost. The profit relation with consumers claiming cost can explain why some firms encourage and some others discourage consumers from complaining. Finally, independently of its pricing strategy, the firm always overprovides product reliability. The fact that the firm cannot fully appropriate the benefits of replacing defected units generates incentives to overprovide product re-
liability. This effect is exacerbated by the fact that many consumers make undesirable complaints, as consumers only internalize their private cost of making complaints. Only in some extreme cases, where the consumers’ heterogeneity disappears, market outcome is efficient.

This article provides the following contributions. First, it endogenizes the number of claimants among buyers as a function of price; note that the pricing strategy can exclude some particular types of consumers from buying and, of course, from complaining. Second it recognizes that policies that encourage consumers to exercise their rights and guarantee the provision of more reliable products may have conflicting goals. Third, it provides rationalization for the fact that some firms encourage consumers to voice their complaints and some others discourage consumers from complaining. Finally, it shows that when it faces heterogeneous consumers in their response of making complaints, the firm have incentives to overprovide product reliability.

We illustrate our results with a very simple example. Assume that there is a mass 1 of consumers that decide whether to buy one unit of a product that fails with probability $x$ (which is learnt after purchased). A proportion $q$ of these consumers are claimants at zero complaining cost and the complement consumers, $1-q$, are non-claimants. A consumer enjoys a utility 2.5 when buying a product that works well and 0 otherwise. In the supply side, one firm maximizes profits choosing the price $p$ and a pair of failure rate $x$ and manufacturing unit cost $c$, i.e., $(x, c) \in \{(0.1, 1); (0.2, 0.7); (0.3, 0.5)\}$, determining the technology of production; it is more costly to manufacture a product with lower failure rate. Complains are always granted and, for simplicity, we assume that the replacing unit always works well. However, a complain implies that the firm incurs in a unit manufacturing cost $c$ and an additional cost of 1 for dealing with the complaint.

The firm’s strategy depends on the proportion $q$ of claimants among consumers. If $q \leq 0.36$ the firm chooses $x = 0.2$ and $c=0.7$ and charges a price $p = 2 (= 2.5 \times 0.8)$ selling to all consumers. If $0.36 < q \leq 0.72$, the firm chooses $x = 0.1$ and $c = 1$ and sells to all consumers at a price of $p = 2.25 (= 2.5 \times 0.9)$. Finally, if $0.72 < q$ the firm changes its strategy and chooses $x = 0.3$ and $p = 2.5$ selling only to claimants, then the demand is equal to $q$. This simple example shows that reducing consumers’ claiming cost, which increases the number of claimants, may generate an increment in product’s failure rate, i.e., from $x = 0.1$ (or $x = 0.2$) to $x = 0.3$. With respect to profit, firm’s profit has a U-shaped relation with $q$: it is decreasing in $q$ for $q < 0.72$ and increasing in $q$ for $q > 0.72$. This result implies that firm’s profit may increase or decrease when the number of claimants increases.

Our results are robust to different specifications. Our results remains if the firm replace defected units with probability $q < 1$ when requested. They also hold if there is heterogeneity in consumer’s valuations and/or in consumer’s valuations. Alternatively, there could exist correlation (1) between consumer’s valuations and consumers claiming cost; or (2) between firm’s cost of receiving complaints and consumers claiming cost. Finally, our results are robust to the case where product failure rate depends on expenditures in product design whereas manufacturing cost is constant and does not change with product failure rate.\(^5\)

\(^5\)Expenditures in product design must be decreasing and convex in product failure rate.
The article is structured as follows. Section 2 reviews the literature. Section 3 introduces the model, explains the firm’s strategies and present the social planner problem. Section 4 states the equilibrium, the main proposition and the welfare analysis. Section 5 describes the robustness of the model. Finally, Section 6 concludes. You find all proofs in the Appendix B.

2 Related Literature

Our article relates to the literature on product liability. Oi (1973) analyzes cases where consumers perfectly observe product reliability and several articles of Daughety and Reinganum (1995, 2005, 2008) and Simon (1981) analyze different cases where product quality is not observed. Oi (1973) analyzes how a change in product’s liability from consumer to producer can affect quality negatively. In his approach consumers are homogeneous and there is no cost of making complaints. Daughety and Reinganum (1995, 2005, 2008) analyze how different situations of the legal environment affects product’s safety. Daughety and Reinganum (1995) study the relation between R&D phase and product’s safety when price may work as a signal of product’s safety under strict liability. Daughety and Reinganum (2005) analyze the relation between safety, R&D and confidential settlements. Daughety and Reinganum (2008) work on the relation between signaling price and costly information disclosure when product’s safety is private information. These authors also assume that consumers are homogeneous.

The article that is closest to us is Simon (1981) who studies the impact of costly litigation and imperfect information about product quality and the outcome of a lawsuit on the existence of negligent firms. In her environment, a reduction in consumers’ litigation cost fosters firms to increase product reliability. She models imperfect information about product quality in consumer side, whereas we introduce heterogeneity in consumer’s claiming cost (or consumer’s valuation), or, alternatively, asymmetric information in firm’s side. More importantly, she bans the possibility of not buying, banning the demand effect.\(^6\)

Our article also relates to the vast literature on warranties as an exploitation device.\(^7\) To cite just a few works, Mann and Wissink (1990), Murthy and Djamaludin (2002) and Huang, Liu, and Murthy (2007) analyze how warranties may affect the firm’s choice of product reliability and demand. In contrast to the evidence that many consumers do not exercise their rights or warranties, these articles assume that it is costless to exercise a warranty or a consumer right. Most results that link warranty conditions and product reliability (even on signaling approach of warranties) relies on the fact that consumers exercise their guarantees and that consumers claiming cost is almost null. An exception is Palfrey and Romer (1983) who stress the relation between optimal warranty and buyer-seller disputes. However, they assume that consumers are homogeneous instead of heterogeneous, and that product reliability is exogenous, which is precisely the issue of our article.

Spence (1975) analyzes the incentives to overprovide or subprovide product quality of a monopoly

---

\(^6\)In Shavell (2007) you find other important aspects of product liability for accidents.

\(^7\)Emons (1989) classifies the literature on warranties according to its purpose in exploitation, to increase consumers willingness to pay, signaling, to reveal product reliability, and incentive, to increase quality conditions. See also Priest (1981) for an investment approach.
comparing the effect of quality on the marginal consumer and on the average consumer. Our welfare analysis relates closely to this paper, extending the model in two dimensions. In our environment, the firm not only analyzes the effect of reliability on the marginal consumer (as in his case), but also it internalizes the effect of reliability on the average cost (which differ for the firm and the social planner). More importantly, the heterogeneity in consumers claiming cost provides multiple strategies to the firm, modifying the incentives to provide reliability.

Belleflamme and Peitz (2007) show that firms may have incentives to overprovide product reliability when investments to improve product reliability are observable and provide a signal of unobservable product quality. Our result, instead, applies even when product reliability is observable, introducing a direct new mechanism that leads to overprovision of product reliability.

There are other mechanisms to motivate a provision of reliability. Klein and Leffler (1981) study the choice of product reliability by the firm in a dynamic framework. Repeated purchases, reputation, and brand name may ensure the provision of high quality goods by the firm. Finally, Greif, Milgrom, and Weingast (1994) describe how merchants create guild to improve the terms of trade, such as the level of product quality/reliability. We work on a static model with perfect information where reputation does not play any role in guaranteeing the provision of product reliability. Our article also relates to the literature on consumers’ complaint behavior (CCB), which focuses on the consumers’ reaction to dissatisfaction (Chebat, Davidow, and Codjovi, 2005; Owens and Hausknecht, 1999).

3 Model

We consider the problem of a monopolist choosing its product’s price $p$ and failure rate $x \in [0, 1]$ to maximize its profit, whereas anticipating that consumers may request a replacement if the product they have purchased is faulty. The firm grants all replacement requests (there is binding legislation) by exchanging faulty products for new ones.\footnote{Our results are robust to situations where replacements are not granted. See Section 5.} Consumers are heterogeneous in their cost of requesting a replacement, which may lead to some consumers scrapping faulty products. Obviously, a replacement itself may fail as well and the consumer may again request a replacement, which will, again, be granted.\footnote{Our model combines the simplicity and the minimum elements that are present in Palfrey and Romer (1983) and Mann and Wissink (1990), adding heterogeneous consumers’ claiming cost.}

Consumers are indexed by $i$ and are heterogeneous in their cost of requesting a replacement, $k_i \in \mathbb{R}_+$. We assume $k$ is distributed in $\mathbb{R}_+$ with cumulative distribution $G(k, \lambda)$, where $\lambda$ characterizes the distribution. $G(k, \lambda_1) \geq G(k, \lambda_2)$ for any $k$ if and only if $\lambda_1 \leq \lambda_2$. We place sufficient assumptions over $G(k, \lambda)$ at the end of Section 3. Consumers derive utility $v \geq 0$ from consuming one unit of a non-faulty good. If the good is faulty or the consumer chooses not to purchase it, the consumer derives a utility of 0. Each consumer $i$ makes two sequential decisions, whether to purchase one unit of the product and whether to pursue a replacement if the product breaks down. The consumer pursues a replacement if $v(1 - x) - k_i \geq 0$. Notice that the consumer’s replacement decision is the same
independent of whether the product being replaced is the original product or a replacement. This implies that the expected utility consumer $i$ derives from replacing faulty products is given by

$$EU_i(p,x) = v(1-x) - p + \frac{x}{1-x} \max \{ v(1-x) - k_i, 0 \} \geq 0.$$  

Equation (2) shows that those consumers who purchase the good and then request replacements are those with lowest $k$, whereas those with highest $k$ prefer to purchase the good and will not replace it if it is faulty. The characterization of the actual consumption choice depends on $p$. If $p$ is high (i.e., $p > v(1-x)$), all consumers who purchase replace the product if it is faulty, with their willingness to pay only depending on $x$ by the expected replacement cost. Instead, if $p$ is low (i.e., $p \leq v(1-x)$), some consumers purchase and replace whereas others purchase and scrap.

We next obtain the demand function. We aggregate the consumers’ choices for all $k$, but separate them into two groups, depending on whether they will request a replacement if the good results to be faulty. Those consumers with $k_i \leq v(1-x)$ anticipate replacing a product if it is faulty. We label them **claimants**. Among these consumers, those with $k_i \leq (v-p)(1-x)/x$ purchase the product. Instead, the consumers with $k_i \geq v(1-x)$ anticipate not replacing a faulty product. We label these **non-claimants**. All consumers who are non-claimants make the same purchasing decision. If $v(1-x) \geq p$, all non-claimants purchase the product, whereas otherwise, all non-claimants do not. (Notice that the difference in thresholds $v(1-x)$ and $(v-p)(1-x)/x$ can be rewritten as $(1-x)(p-v(1-x))/x$, which implies that the demand cut-offs are solely determined by the comparison of $p$ and $v(1-x)$.) That is, the demand function of claimants, $Q^C$, and non-claimants, $Q^{NC}$, is given, respectively, by

$$Q^C = \begin{cases} G\left(\frac{(v-p)(1-x)}{x}, \lambda \right) & \text{if } p > v(1-x), \text{ and} \\ G(v(1-x), \lambda) & \text{if } p \leq v(1-x). \end{cases}$$

$$Q^{NC} = \begin{cases} 0 & \text{if } p > v(1-x), \text{ and} \\ 1 - G(v(1-x), \lambda) & \text{if } p \leq v(1-x). \end{cases}$$

Demands shape will be inherited by the form of $G(k, \lambda)$. Notice that the effect of $x$ on the expected utility of buying (in Equation (2)) is $-\frac{x}{(1-x)^2}$ for claimants, which decreases as $k$ decreases, and is $-v$ for non-claimants (the same for all). The effect of $x$ to demand functions follows the same reasoning. Now, we turn to the firm’s problem.

**Monopolist’s Problem.** We let $c(x)$ denote the marginal cost of production, which is constant for a given $x$. We assume $c(x) > 0$, $c'(x) < 0$ and $c''(x) > 0$. We also assume that $xc(x)$ is weakly increasing in $x$. This assumption implies that $xc(x)/(1-x)$, i.e., the expected cost of manufacturing replacement units, is increasing in the failure rate.$^{10}$ When serving replacement requests, the firm also incurs a per-
unit replacement cost \( c_r \geq 0 \). This implies that the cost of selling one unit of the good to a claimant consumer is \( c(x) + x(c(x) + c_r)/(1-x) \), whereas selling the same unit to a non-claimant is only \( c(x) \).

Then, given the demand function in (3), the monopolist’s maximization problem is given by

\[
\max_{(p, x)} \Pi(p, x) = \begin{cases} 
  p - c(x) - x c_r/(1-x) G \left( \frac{(v-p)(1-x)}{x}, \lambda \right) & \text{if } p > v(1-x), \text{ and} \\
  p - c(x) - x c_r/(1-x) G \left( v(1-x), \lambda \right) & \text{if } p \leq v(1-x).
\end{cases}
\]

For the problem to be non-trivial we make the following assumption:

\( A1 \) - \( v(1-x) - c(x) > 0 \) for some \( x \in [0, 1] \).

Assumption A1 implies that it is socially optimal that all consumers are served. It also ensures that, for a range of \( c_r \), there exists an equilibrium where the firm makes positive profits.

**Lemma 1.** Under assumption A1 there exists \( \tau_r \) which is implicitly defined by \( \max_x (v - c(x) - x c_r/(1-x)) = 0 \) such that, if \( c_r < \tau_r \), the firm makes positive profits for any \( G(k, \lambda) \) with full support.\(^{12}\)

\( A2 \) - \( c_r < \tau_r \).

Assumption A2 guarantees that there always exists an equilibrium where the firm makes positive profits.\(^{13}\)

Assumption A1 implies that it is efficient that the firm sells to all consumers, i.e., choosing \( p \) and \( x \) such that \( p \leq v(1-x) \). However, as the firm cannot appropriate all the value generated to consumers, the firm may have incentives to exclude some consumers from the market, i.e., choosing \( p \) and \( x \) such that \( p > v(1-x) \). We say that the firm follows a high-pricing strategy if \( p \) and \( x \) are such that \( p > v(1-x) \); otherwise, we say that the firm follows a low-pricing strategy. Notice that \( p = v(1-x) \) dominates all other \( p \) and \( x \) that constitute a low-pricing strategy as the objective function depends positively on \( p \) for \( p \leq v(1-x) \).

The firm faces a trade-off in its choice of price and failure rate which are quite different under each strategy. When following a low-pricing strategy the marginal consumer is a non-claimant whose willingness to pay depends only on the probability of buying a non-defected unit; reducing the failure rate increases considerably his willingness to pay. Modifying the willingness to pay of this marginal consumer allows the firm to increase the price charged for all units sold. Additionally, some consumers are claimants and the cost associated with replacements decreases as the failure rate decreases. Reducing the failure rate obviously increases the manufacturing cost of all units sold.

---

\(^{11}\)The parameter \( c_r \) may account for a variety of costs, from administrative/shipping and handling costs to reputational losses.

\(^{12}\)If additionally \( c_r \leq \tau_r \), where \( \tau_r \) is implicitly defined by \( \max_x (v - c(x) - x (c(x) + c_r)/(1-x)) = 0 \), then for any \( G(k, \lambda) \) both it is socially optimal that all consumers are served and the firm ensures positive profits selling to all consumers in the market.

\(^{13}\)If \( c_r > \tau_r \), there exists \( \lambda_0(c_r) \) such that the firm sells no unit if and only if \( \lambda < \lambda_0(c_r) \). A lot of consumers are willing to initiate inefficient complaints (see welfare section) generating a lot of replacement cost such that the firm prefers not to sell. If \( c_r > \tau_r \), the firm serves to all consumers if and only if consumers claiming cost is sufficiently high.
In contrast, when following a high-pricing strategy the marginal consumer is a claimant whose willingness to pay depends both on the failure rate and on his own claiming cost. Reducing the failure rate increases his willingness to pay but this increment decreases as his claiming cost decreases. The marginal cost of selling this unit incorporates a manufacturing cost and a replacement cost, the former cost increases and the latter decreases as the failure rate decreases.\footnote{The unit cost that incorporates the manufacturing and replacement costs has a U-shaped function of product’s failure rate, i.e., \(c(x) + x(c(x) + c_r)/(1 - x)\) has a U-shaped function of \(x\). Firm always chooses a failure rate in the decreasing part.}

Welfare. For a welfare analysis we first identify the sources that prevent the firm to achieve the first best. There are two sources of inefficiencies in this model: 1- the firm cannot appropriate all the value that generates to consumers, modifying the failure rate and the price to maximize its profits; 2- consumers only internalize their private cost of making complaints, generating more complaints than socially optimal.\footnote{The social benefit of a complaint is \(v(1 - x)\) and the social cost of a complaint is \(c(x) + c_r + k\). A consumer only internalizes his/her private cost \(k\), consequently consumers with \(k \in [v(1 - x) - c(x) + c_r, v(1 - x)]\) generate inefficient complaints.} Both of these inefficiencies are recognized in the literature, the first inefficiency in the literature of asymmetric information (Matthews and Moore, 1987) and the second one in the literature of law and economics (Shavell, 2007). The following lemma introduces the welfare function for a given market outcome (i.e., for a given failure rate \(x\), demand function and consumers complain behavior when receiving a defected unit).

**Lemma 2.** The welfare of an economy, where product’s failure rate is \(x\), where consumers with type \(k \leq k_B\) buy the product, and where consumers with type \(k \leq k_C\) complain if they receive a defected unit, is represented by the following expression:\footnote{Notice that, by construction, \(k_C \leq k_B\), i.e., a consumer can ask a product replacement only if he has bought it.}

\[
W = [v(1 - x) - c(x)]G(k_B, \lambda) + x \left[ v - \frac{c(x) + c_r + E[k|k \leq k_C]}{1 - x} \right] G(k_C, \lambda) \tag{3}
\]

The first term represents the welfare generated by selling the product and the second term stands for the welfare generated by replacing defected units. Under a high-pricing strategy \(k_C = k_B = k\), where \(k\) is the marginal consumer type chosen by the firm; under a low-pricing strategy \(k_C = v(1 - x)\) and \(k_B = +\infty\); and, a social planner chooses \(k_C = \max\{v(1 - x) - c(x) - c_r, 0\}\) (see footnote 15) and \(k_B = +\infty\). Notice that if \(v < \frac{c(x) + c_r}{1 - x}\), \(k_C = 0\), that is no complain is socially desirable.

Assumptions over \(G(k, \lambda)\). In order to guarantee sufficient conditions for our results we assume that the distribution function \(G(k, \lambda)\) is twice continuously differentiable with respect to \(k\) and continuously differentiable with respect to \(\lambda\). We require that \(G(k, \lambda)\) satisfies the increasing and convex reverse hazard rate property in \(k\) for any \(\lambda\), to guarantee that the equilibrium under a high-pricing strategy is characterized by the first order conditions of the firm’s problem.

For the sake of the comparative static respect to \(\lambda\) (i.e., consumers claiming cost distributions moving to the left), we work with any family of distributions that can be ordered with respect to the reverse hazard rate dominance. A distribution \(G(k, \lambda)\) is dominated in terms of reverse hazard rate by another
distribution $G(k, \lambda')$ if and only if $\lambda < \lambda'$. Finally, we assume that $\frac{\partial G(k, \lambda)}{\partial \lambda} - k \frac{\partial g(k, \lambda)}{\partial \lambda} < 0$ for all $k > 0$ to guarantee that, when following a low-pricing strategy, firm’s profit is supermodular with respect to $x$ and $\lambda$.\footnote{All assumptions in this section are sufficient but not necessary for our results to hold. Nevertheless, the exponential distribution ($G(k, \lambda) = 1 - e^{-k\lambda}$) satisfies all assumptions required here.}

4 Equilibrium

We next find the monopolist’s optimal strategy for a given distribution of consumers’ claiming cost and then analyze the effect of a reduction in the consumers’ claiming cost, i.e., the effect of a reduction in $\lambda$.\footnote{We look for Subgame Perfect Nash Equilibriums.}

The firm’s optimal choice of $p$ and $x$ depends crucially on the parameter $\lambda$ which is summarized in the following proposition.

Proposition 1. If consumers claiming cost is high the firm follows a low-pricing strategy and if consumers claiming cost is low the firm follows a high-pricing strategy. The firm’s profit function has a U-shaped relation with consumers claiming cost.

Formally, there exists a cut-off in the parameter value characterizing the consumers’ claiming cost distribution, $\hat{\lambda}$, such that the firm follows a low-pricing strategy if $\lambda \geq \hat{\lambda}$ and follows a high-pricing strategy otherwise. The firm’s profit function has a U-shaped relation with $\lambda$, with a minimum value at the cutoff $\hat{\lambda}$.

The intuition is as follows. If consumers claiming cost is high, i.e., high $\lambda$, there are many non-claimants among consumers. The firm has incentives to reduce the price increasing demand and revenue, and thus to follow a low-pricing strategy. Under a low-pricing strategy the firm bears a cost, but receives no benefit, of replacing defected units to claimants. The firm has incentives to increase product reliability in order to increase the willingness to pay of non-claimants (and consequently the price) and to reduce the likelihood of receiving complaints. The second effect increases, and firm’s profit decreases, as consumers claiming cost shift down, i.e., as $\lambda$ decreases.

If consumers claiming cost is low, i.e., low $\lambda$, there are many claimants among consumers. If his claiming cost goes down, a claimant’s expected utility from complaining will increase, and consequently it will increase his willingness to pay for the product. If this is the case, the firm has incentives to follow a high-pricing strategy in order to partially appropriate the benefit of replacing defected units. The additional surplus appropriated per unit sold compensate the lower profit for excluding some consumers from buying. The higher the price, the higher the benefit appropriated to buyers but the lower the demand. The firm has incentives to provide product reliability in order to increase consumers’ willingness to pay (increasing both price and demand) and to reduce the likelihood of receiving complaints. The first effect decreases (reducing reliability), and firm’s profits increases, as consumers claiming cost go down.
Proposition 1 provides a rationale for the firm’s incentives to encourage or discourage consumers of asking for a product replacement. If the firm follows a low-pricing strategy, its profit increases when consumers have higher claiming costs. If this is the case, the firm would be willing to affect consumers’ claiming costs upward even when this action is costly. That is, the firm would have clear incentives to provide a tiresome procedure for requiring its customer service to discourage consumers from complaining. If, instead, the firm follows a high-pricing strategy, we observe the opposite result. The firm’s profit decreases with consumers’ claiming cost and the firm would provide devices to aid consumer complaints, even when it is costly.

We argue in the introduction that the empirical evidence shows that many buyers do not request a replacement despite having received a defective product. In our environment, this evidence is consistent with the firm following a low-pricing strategy so that non-claimants also purchase the product. In Proposition 2, we show that, if the firm’s cost of receiving complaints is not too high, the firm may reduce its product’s reliability when the consumers’ claiming cost is reduced.

**Proposition 2.** If the firm’s liability cost of receiving complaints is not too high, there exists a reduction in the consumers’ claiming cost to which the firm responds by reducing the level of its product’s reliability.

Formally, if \( c_r < \max_x v(1-x) - c(x) \), for any \( \lambda \) there exists \( \lambda' \), lower than \( \lambda \), such that \( x(\lambda') > x(\lambda) \).

The intuition of this result is as follows. If consumers claiming cost is low, the firm will follow a high-pricing strategy charging a high price that reduces demand. Increasing product reliability decreases the expected number of replacements, and increases both consumers’ willingness to pay, allowing to charge a higher price, and demand. Note that consumers’ willingness to pay internalizes the expected future cost associated to replacements. Replacements are less likely if the product is more reliable and they are less costly if complaints are cheaper. Consequently, the effect of product reliability on willingness to pay and on demand diminish as consumers claiming cost decreases. Then, product reliability decreases as consumers claiming cost decreases.

If consumers claiming cost is high, the firm will follow a low-pricing strategy. In this case, demand is high and the price is limited by the willingness to pay of the marginal consumer who does not find rewardable replacing defected units. However, there are some consumers, defined as claimants, that always replace defected units. Increasing product reliability reduces the expected number of replacements and increases the willingness to pay of the marginal consumer. The number of claimants and the incentives to provide product reliability increase as consumers claiming cost decreases. In this case, a marginal reduction in consumers claiming cost increases the number of claimants and product reliability, and reduces firm’s profit (which coincide with common wisdom). However, the more the claimants the higher the surplus consumers make by replacing defected units. The firm may change from low to high-pricing strategy if there is a discrete reduction in consumers claiming cost. This change modifies the incentives to provide product reliability as described in the previous paragraph, and, eventually, product reliability can decrease as consumers claiming cost decreases.
Proposition 2 states a clear result: a discrete reduction in consumers’ claiming cost might generate a reduction in product’s reliability and an increase in firm’s profit. The policy implication of Proposition 2 is that any policy that pursues to help consumers to file complaints and to increase product reliability may have conflicting goals.

This result, that product reliability can decrease when there is a reduction in consumers’ claiming cost, is more likely when the firm has a low cost of receiving complaints. However, it may also takes place for high values of the firm’s liability cost. Under a high-pricing strategy, the effect on replacements is what motivates the firm to increase product reliability. Whereas the effect on demand is only determined by consumers’ willingness to pay, the effect on replacements is determined by the firm’s cost of facing complaints. When the firm has a low liability cost, it has more incentives to reduce reliability if consumer’ claiming cost decreases.\textsuperscript{19}

Welfare Analysis. We now show that the firm provides products with lower failure rate than a social planner would choose. We analyze the consequences of the two sources of inefficiencies described in Section 3 and, then, we describe the effects of a reduction of consumers’ claiming cost on welfare.

Because it does not fully internalize the benefits generated to consumers of providing product reliability, the firm chooses an inefficient level of it. In order to increase the surplus appropriated in the market the firm overprovides product reliability. In doing so, it increases the willingness to pay of the marginal consumer and the price, and minimizes the expected cost associated with replacements. This effect holds under both a low-pricing and high-pricing strategies. The high-pricing strategy implies also that the firm excludes some consumers from buying, reducing welfare.

The fact that many buyers make inefficient complaints exacerbates the overprovision of product reliability.\textsuperscript{20} These inefficient complaints increase the firm’s cost associated with replacements. As a consequence, the firm has incentives to reduce the failure rate in order to lower the expected number of complaints. The following proposition summarizes these results.

\textbf{Proposition 3.} If the distribution of consumers claiming cost has full support, the market outcome leads to an overprovision of product’s reliability, i.e., the failure rate is lower than the optimal level.

In extreme cases, both when consumers claiming cost tends to infinity or to zero, the firm can appropriate the value generated to consumers. Their complaining behavior coincides with the socially desirable in both cases if \( c_r < \max v(1-x) - c(x) \). Consequently, in these extreme cases the market outcome is efficient.

Finally, a reduction in consumers claiming cost have positive and negative effects on welfare, leaving uncertain the net effect. Under a low-pricing strategy, a reduction in consumers claiming cost will allow more consumers to complain. As consumers only internalize their private cost of complaining, some of

\textsuperscript{19}If \( \max v(1-x) - c(x) < c_r < \tau_r \), there exists \( \lambda(c_r) \) such that our result holds for any \( \lambda < \lambda(c_r) \). That is, if \( \lambda < \lambda(c_r) \), there exists \( \lambda' \) lower than \( \lambda \) such that \( x(\lambda') > x(\lambda) \). In this case complaints are not socially desirable. If it were possible to refrain consumers from complaining the inefficiency is resolved.

\textsuperscript{20}If all complaints are inefficient, the overprovision of reliability is generated by consumers. If this is the case, banning complaints allows the firm to provide the optimal level of reliability.
these complaints are desirable but some are not. The net effect will depend on how many of “desirable” and “non-desirable” complaints realize. A second (negative) effect on welfare takes places when the firm switches strategy from low to high-pricing due to a reduction in consumers claiming cost. This change in firm’s strategy reduces the number of buyers and consequently reduces welfare. A third (positive) effect takes place when the firm is following a high-pricing strategy. Under this strategy a claiming cost can be interpreted as a transaction cost. Any reduction in consumers claiming cost increases welfare through lower transaction cost and through higher demand (as some consumers find now rewardable to buy the product).

5 Robustness

We develop the simplest model to shed light on our contribution. Nevertheless, our results also holds for different specifications.

We assume that replacements are always granted. Assuming that, when requested, a replacement takes place with probability \( q \) \((< 1)\) does not change our results. Consistent with our results, the firm’s profit is increasing in \( q \) under a high-pricing strategy (i.e., when consumers claiming cost is low) and it is decreasing in \( q \) under a low-pricing strategy (i.e., when consumers claiming cost is high). If possible, the firm would help consumers to complain and would process claims with higher probability when consumers claiming cost is low, and vice versa.

We assume that it is more expensive to manufacture a more reliable product, i.e., \( c'(x) < 0 \). Our results also hold if we assume that, instead, the firm invests in product design (e.g., R&D) to develop a more reliable product whereas its manufacturing cost is constant for any failure rate, e.g., an innovation about how to assemble the product. It is enough to assume that product design expenditures are increasing and convex in product reliability for our results to hold. A new effect naturally arises: product design expenditures are a sunk cost, thus the per unit sunk cost is lower when demand is higher.

It is likely that the Information and Communication Technologies (ICT) like internet had decreased not only consumers claiming cost but also the firm’s cost of dealing with complaints. A positive correlation between these costs emphasizes our results: if both costs are low there is a lot of claimants among buyers and the firm incurs in a low cost of receiving complaints. The incentives to provide reliability decrease both because replacements are not too costly and because product reliability has a low effect on consumers willingness to pay. Consequently, reliability is quite low under a high-pricing strategy. Analogously, reliability is quite high under a low-pricing strategy, and our results are more likely.

In the model we assume that consumers’ valuation of the product are the same across consumers. In the market, consumers may be heterogeneous in their valuations and claiming cost. These two dimensions may also be correlated with product valuation. For instance, a consumer with higher income may have both higher monetary valuation for a product and higher oppotunity cost of time (which is necessary to request a replacement). We consider both the case where the consumers valuation and claiming cost are heterogeneous among consumers, and the case where these two dimensions are correlated, i.e.,
the claiming cost is a fix proportion of consumers valuation, and our results still holds. As the time to request is proportional to the whole endowment of time that individuals have, this assumption looks reasonable. Moreover, consumers with higher income can buy others people time or a legal insurance to handle with these complaints. Finally, note that wealth and education are positively correlated, and that well educated individuals usually know better the legal procedure to make a complaint than less educated consumers, implying that these consumers may complain incurring in less time than consumers with lower income.

6 Conclusion

Consumers’ associations goals are, among others, to help consumers voice their complaints and enforce the provision of reliable products. In this article, we show that these goals may not be aligned: the firm may decide to produce a less reliable product if more consumers request a replacement of a defected product.22

The main contribution in terms of policy implication of this article is to warn policy makers that reliability is an endogenous decision. Product reliability depends on both consumers’ claiming costs and firm’s replacement costs. An increase in consumers’ complaints may be followed by a reduction in product’s reliability, which in turn generates additional complaints. Then, observing a sharp increment in complaints is not necessarily a good market signal. This article also points out that the expected number of complaints is endogenously determined by the price and product reliability.

Having difficulties to measure this effect empirically does not diminish the relevance of our results. The fact that many companies are turning to encourage consumers to complain can provide ideal situations to test our results.23

Concluding, this article complements the literature on product liability and warranties pointing out that, when consumers have heterogenous claiming cost, firms may have incentives to overprovide product reliability, and this inefficiency is exacerbated by inefficient consumers complaints.

References


21 For instance, a consumers associations can provide a consumer help service for a monthly fee. They have a thorough legal knowledge and they can lobby firms to enforce a fair consumer request.

22 The implication for consumers associations is that to maximize consumer’s surplus there is an optimal level of complaining behavior (complaints). Below this level firms would change their strategies in order to appropriate part of consumer’s surplus.

23 Several websites, like getsatisfaction.com, are used by thousands of firms to handle complaints efficiently and mediate between consumers and firm.


A Appendix

Solving firm's problem for High-Pricing Strategy and Low-Pricing Strategy

Under high-pricing strategy, i.e., \( p > v(1 - x) \), we make a simple transformation: we define \( k = \frac{(v - p)(1 - x)}{x} \) where \( p = v - \frac{xk}{1-x} \). Because \( p \in [v(1-x), v] \subset \mathbb{R}_+ \) and \( x \in (0,1) \), then \( k \in [0, v(1-x)] \subset \mathbb{R}_+ \).

The problem is re-expressed as \( \max_{k,x} \Pi(k,x) = \left[ v - \frac{c(x) + x\lambda}{1-x} \right] \Pi(k,\lambda) \). This expression allows us to find, first, the optimal choice of \( x \) as a function of \( k \), and second, the value of \( k \) that maximizes firm’s profit. For any \( k \), there exists \( x(k) = \arg\min \left[ \frac{c(x) + x\lambda}{1-x} \right] \) with \( x'(k) < 0 \), and \( G(k,\lambda) \) is increasing in \( k \). The value of \( k \) is defined by the first order condition,

\[
\frac{[v(1-x) - c(x)]}{x} - c_r = \frac{G(k,\lambda)}{g(k,\lambda)} + k. \tag{4}
\]

For \( k = 0 \) the left hand side is positive and the right hand side is zero. Both, the left hand side and the right hand side, are increasing in \( k \). The derivative of left hand side is \( -\frac{2x}{xk} + \frac{v-c(x) + xc'(x)}{x^2} \), which is positive as \( \frac{2x}{xk} < 0 \) and \( v - c(x) + xc'(x) = v - c(x) - x\frac{c(x) + c_r}{1-x} \). If \( \frac{2x}{xk} > 0 \) (which holds if \( c''(x) \) is negative or not too high) and if \( v - c(x) + xc'(x) - 0.5x^2c''(x) > 0 \), the left hand side is concave. \(^{24}\)

On the contrary, the derivative of the right hand side is \( \frac{3}{xk} \frac{G(k,\lambda)}{g(k,\lambda)} + 1 \). Assuming that the reverse hazard rate (i.e., \( RHR = \frac{g(k,\lambda)}{g(k,\lambda)} \)) is decreasing and concave in \( k \) guarantees that the solution is unique. If \( G(k,\lambda) \) is log concave, then the reverse hazard rate is decreasing. Finally, the second order condition respect to \( k \) is \( \Pi_{kk} = -\frac{2x}{xk} g(k,\lambda) - \frac{1}{(1-x)^2} \frac{\partial}{\partial k} G(k,\lambda) + g_k(k,\lambda) \left[ v - c(x) - x\frac{c(x) + c_r}{1-x} \right] \). For the second order condition to be negative we need that \( g_k \) to be negative or not too positive. We also require that \( \frac{\partial}{\partial k} = \frac{-1}{c''(x)(1-x)} \) to be not too high, which is guaranteed if \( c(x) \) is sufficiently convex.

For the low-pricing strategy the profit function is \( \Pi(x) = v(1-x) - c(x) - x\frac{c(x) + c_r}{1-x} G(v(1-x),\lambda) \). The solution is unique if there is no mass points (particularly, around the marginal consumer). The second order condition is \( \Pi_{xx} = -c''(x) - v^2x\frac{c(x) + c_r}{1-x} g_k + 2v \left[ \frac{c''(x)}{1-x} + \frac{c(x) + c_r}{(1-x)^2} \right] g - \left[ x\frac{c''(x)}{1-x} + \frac{2c'(x)}{1-x} + \frac{2c(x) + c_r}{(1-x)^2} \right] G(.) \), that is negative if \( c(x) \) is sufficiently convex.

B Appendix: proofs

Proof of Lemma 1

Proof. For \( c_r < \bar{c}_r \), let \( k^r = \frac{2-v}{2} > 0 \). Because the distribution \( G(.) \) has full support, \( G(k^r,\lambda) > 0 \).

Following the solution in Appendix A for a high-pricing strategy and by construction, \( c_r + k^r < \bar{c}_r \).

\(^{24}\)The second derivative of the left hand side (LHS) is

\[
\frac{\partial^2 \text{LHS}}{\partial k^2} = \frac{\partial}{\partial x} \left[ \frac{2c''(x)}{x^3} - \frac{v-c(x) - xc'(x)}{x^3} \right] - \frac{v-c(x) - xc'(x)}{x^2} \frac{\partial^2 x}{\partial k^2} < 0.
\]
Then, we can guarantee \( \max_{x} p - c(x) - x \frac{c(x) + c_r + k_r}{1 - x} > 0 \) and the firm makes positive profit. \( \square \)

**Proof of Proposition 1**

**Proof.** For any \( 0 < \lambda < +\infty \), the profit function is discontinuous at \( p = v(1 - x) \). However, we can consider the expressions under a low-pricing strategy and under a high-pricing strategy separately. By the envelope theorem \( \Pi(p) = v(1 - x) \) is increasing in \( \lambda \) and \( \Pi(p > v(1 - x), \lambda) \) is decreasing in \( \lambda \). For details on pricing strategies see Appendix A.

Note that if \( \lambda \to +\infty \) there is only non-claimants and \( \Pi(p = v(1 - x)) > 0 = \Pi(p > v(1 - x)) \). If \( \lambda \to 0 \) there is only claimants and \( \Pi(p = v(1 - x)) < \Pi(p = v - \varepsilon) \), for some \( \varepsilon > 0 \). For any \( \lambda \) there exists an equilibrium with positive profits that is characterized by at least one of these two strategies. The existence of a cutoff \( \hat{\lambda} \) such that \( \Pi(p = v(1 - x), \hat{\lambda}) = \Pi(p > v(1 - x), \hat{\lambda}) \) is guaranteed. \( \square \)

**Proof of Proposition 2**

**Proof.** We show that \( x \) is an increasing function of \( \lambda \) when the firm follows a low-pricing strategy, i.e., \( p = v(1 - x) \), and \( x \) is a decreasing function of \( \hat{\lambda} \) when the firm follows a high-pricing strategy, i.e., \( p > v(1 - x) \). Finally, we guarantee conditions for \( x(\lambda \to +\infty) \) under a low-pricing strategy being lower than \( x(\lambda \to 0) \) under a high-pricing strategy, such that there exists a reduction in \( \hat{\lambda} \) (i.e., a reduction in consumers claiming cost) that generates an increase in the failure rate.

Assuming that \( xc(x) \) is non-decreasing in \( x \) and that \( G_\lambda(k, \hat{\lambda}) - kg_\lambda(k, \hat{\lambda}) < 0 \) for \( k > 0 \) is sufficient to guarantee that the objective function is supermodular in \( x \) and \( \lambda \) when the firm follows a low-pricing strategy, i.e., \( \Pi_{\lambda x} = - \frac{c(x) + c'(x) + c_g}{1 - x} [G_\lambda - x(1 - x)g_\lambda] > 0 \). The failure rate \( x \) is increasing in \( \lambda \) when the firm follows a low-pricing strategy.

For a high-pricing strategy, we state the monotonicity of \( x \) respect to \( \lambda \) from two conditions: i) the first order condition with respect to \( k \) in equation (4), where \( \frac{dk}{dx} > 0 \); and ii) the monotonicity \( x'(k) < 0 \). The left hand side of equation (4) does not depend on \( \lambda \). The right hand side in equation (4) decreases in \( \lambda \) as the reverse hazard rate increases when \( \hat{\lambda} \) increases, i.e., \( \frac{dG/G}{d\lambda} > 0 \) implies \( \frac{dG/G}{d\lambda} < 0 \). Any reduction in \( \lambda \) decreases the reverse hazard rate and generates a reduction in \( k \) and thus an increase in \( x \).

If \( \lambda \to 0 \), the firm chooses \( k \to 0 \), \( p = v - \varepsilon \) for some \( \varepsilon > 0 \), and the failure rate \( x_a \) from \( -c'(x_a) = \frac{c(x_a) + c_g}{1 - x_a} \). If \( \lambda \to +\infty \), the firm chooses \( p = (1 - x)v \) and the failure rate \( x_b \) from \( -c'(x_b) = v \). \( x_a \leq x_b \) if and only if \( \frac{c(x_a) + c_g}{1 - x_a} \geq v \). There exists \( \hat{c}_r(v) \in \mathbb{R}_+ \) such that \( \frac{c(x_a) + \hat{c}_r(v)}{1 - x_a} = v \) for which \( x_a = x_b \). Then, \( x_a < x_b \) if and only if \( c_r > \hat{c}_r \). The threshold \( \hat{c}_r(v) \) is increasing in \( v \).

Concluding, the product’s failure rate has a U-shaped relation with \( \lambda \), with a jump at \( \hat{\lambda} \) (\( \hat{\lambda} \) was defined in Appendix A). Finally, it is guaranteed for \( c_r < \hat{c}_r(v) \) that there exists a reduction in consumers claiming cost, i.e., \( \Delta \lambda < 0 \), such that product reliability decreases, i.e., \( \frac{\Delta \hat{c}}{\Delta \lambda} < 0 \). The proof is complete. \( \square \)

**Proof of Proposition 3**

**Proof.** If \( k = v(1 - x) - c(x) - c_r < 0 \) no claimant is socially desirable and \( W = v(1 - x) - c(x) \). The optimal failure rate is \( x_b = \arg \max v(1 - x) - c(x) \). If \( k = v(1 - x) - c(x) - c_r > 0 \), some claimants are
when chooses much as the firm’s choice. Firm’s demand depends crucially on the marginal consumer (which is over (or increases the marginal consumer) significantly, consequently it reduces the failure rate $G$ high or if $v x$ must cross, at least, twice the weighted average (i.e., the optimal failure rate being lower than the firm’s choice of failure rate, the firm’s marginal consumer higher than this weighted average when $x$. Under a high-pricing strategy, $b x = [k + c(x) + c_r + E[k|k \leq \hat{k}]] G(\hat{k}, \lambda)$, the last two terms cancel out. Expressing $v = \frac{\hat{k} + c(x) + c_r}{1 - x}$, we can write the first order condition as $\frac{\partial W}{\partial x} = \left[-c'(x) - \frac{c(x) + c_r + E[k|k \leq \hat{k}]}{1 - x}\right] \frac{G(\hat{k}, \lambda)}{1 - x} = 0$.

The difference in the failure rate values comes from comparing the marginal consumer, $k$, in the firm’s problem with a weighted average of $\hat{k}$ (marginal claimant) and $E[k|k \leq \hat{k}]$ (average claimant) in the social planner problem, i.e., $k_{av} \equiv \frac{\hat{k}[1 - G(\hat{k}, \lambda)] + E[k|k \leq \hat{k}]}{1 - x} G(\hat{k}, \lambda)$. We showed that the marginal consumer $k$ is higher than this weighted average when $\lambda \rightarrow 0$ and when $\lambda$ is high enough (such that $k \geq \hat{k}$). For having the optimal failure rate being lower than the firm’s choice of failure rate, the firm’s marginal consumer $k$ must cross, at least, twice the weighted average (i.e., $k_{av}$), which is not possible given our assumptions over $G(k, \lambda)$. □