Financial Contracting with an Informed Investor*

Eloïc Peyrache†  Lucía Quesada‡

February, 2010

Abstract

We provide a simple model with a privately informed principal who provides financing to a project in which an agent exerts effort that affects the probability of success. When restricting the space of contracts to debt contracts, we show that the equilibrium that gives the highest profit to the principal is a pooling equilibrium in which the allocation of cash-flow rights and the effort of the entrepreneur are independent of the principal’s private information. We then analyze whether the principal, given her informational advantage will have a tendency to be too aggressive or too conservative in her financing policy. In this respect we show that in the best equilibrium the set of types for which there is abusive lending is non-empty if and only if there is over-lending. We then extend the analysis to an unrestricted set of contracts and show that, under some mild conditions, the results above are robust.

JEL Codes: D82, G24, G32.

Key Words: Signaling and incentives, Abusive lending, Inefficient lending.

---

*We thank Leandro Arozamena, Catherine Casamatta, Emilio Espino, Daniel Heymann, Enrique Kawamura, Patrick Rey, Jean Tirole, Federico Weischedelbaum and seminar participants at Université de Toulouse, Universidad Torcuato Di Tella, INSEAD, 1st meeting of Economic Theory of Buenos Aires, LAMES 2008-Rio de Janeiro, FIEL and Universidad de San Andrés. All remaining errors are ours.

†HEC School of Management, Paris, Finance and Economics Department, 1, rue de la Libération - 78351, Jouy en Josas Cedex, France. E-mail: peyrache@hec.fr

‡Universidad Torcuato Di Tella, Department of Economics, Sáenz Valiente 1010, C1428BBI Buenos Aires, Argentina. E-mail: lquesada@utdt.edu
1 Introduction

The aim of this paper is to study the optimal contract between an informed principal who provides financing to a project and an agent who exerts effort that affects the probability of success. Many real life examples illustrate our approach. To fix ideas, let us take a corporate finance perspective, where an investor (the principal) invests in a project and the entrepreneur (the agent) exerts effort. The difficulties of entrepreneurs to obtain financing have been at the center of many contributions. Among others, the uncertainty regarding the profitability of projects, moral hazard issues or overoptimism have been largely explored. We adopt a complementary perspective and follow a recent strand of the literature in corporate finance that stresses the fact that investors may have better information than entrepreneurs regarding some aspects that have an impact on the probability of success of a given project.\footnote{See Garmaise (2007), Inderst and Mueller (2006) and Kaplan and Strömberg (2004).} Venture capitalists who build specific knowledge from focusing on specific sectors or from repeated investment are certainly obvious examples of investors endowed with more information regarding the probability of success of the project than entrepreneurs themselves.\footnote{See Gompers (1996), Hsu (2004) and Kaplan and Schoar (2005).} However, banks can also play such a role when financing small projects. Of particular interest for our purpose is the fact that, following Engel and McCoy (2005), \textit{"the market incentives that historically led lenders to engage in credit rationing have given way to a market where lenders can profit from exploiting new information asymmetries to the detriment of unsophisticated borrowers"}. Finally, some joint-venture contracts, such as those between producers and artists in the entertainment industries, can be discussed along the lines of this paper. In all cases, asymmetric information gives principals an informational advantage when offering their contracts. The focus of this paper is precisely to investigate the impact of such advantage in a setting of bilateral asymmetric information.

We address a basic (financial) contracting problem that relies on these two dimensions. That is, we consider an agent (possibly an entrepreneur) who brings effort to a project but is not endowed with enough wealth to finance it on his own. Financing is then provided by a principal (say an investor) who, in addition to resources and given his own experience, has better information about some parameters that inherently affect the probability of success of the project. To be precise, we consider a bilateral asymmetric information framework where the probability that the project be successful depends on some technological/market parameter only known to the principal and some costly decision (effort, human capital investment) which is only observable by the agent.

Our first purpose in this paper is to analyze the allocation over cash-flow rights as a function of the information owned by the principal. As it is standard in signaling games, there are multiple equilibria
and, unfortunately, usual refinements are (generally) not enough to select only one equilibrium contract. However, we show that the equilibrium that gives the highest profit to the principal is a pooling equilibrium in which the allocation of cash-flow rights and the effort of the entrepreneur are independent of the principal’s private information. The second purpose of the paper is to analyze whether the principal, given her informational advantage will have a tendency to be too aggressive or too conservative in her financing policy. Following Iderst (2008), we define financing by the investor to be too aggressive whenever approval by the financier will result in a negative expected utility for the agent conditional on the principal’s information.

The contribution of our paper is twofold. First, we show that it can be in the principal’s best interest not to transmit her information to the agent even if a better information implies a more efficient effort decision. Second, we obtain aggressive behavior by the investor as an equilibrium outcome of the game. Importantly, these two results hinge on the assumption that the effort of the agent and the information of the principal are complementary. Indeed, within the same structural model, when the information of the principal has no effect on the marginal productivity of effort, none of the statements above remains valid. However, maintaining the assumption of complementarity and adding some assumptions on the distribution function of the principal’s information, we show that the results are still valid when we consider an unrestricted set of contracts.

1.1 Related Literature

Two different strands of literature analyze the possible inefficiencies in contracting with banks. The first one starts with the seminal paper of Stiglitz and Weiss (1981) who studies the adverse selection problem that arises when banks are unable to distinguish high from low quality projects and must offer the same financing to all. In their setting, too few project are financed, namely the high-risk ones. de Meza and Webb (1987) build on Stiglitz and Weiss (1981) but introduce the crucial difference that the expected return on projects differs between entrepreneurs. In a setting where banks know the distribution of the characteristics of the population of entrepreneurs but not the true probability of success, they show that there would be over-investment in low-probability projects.

The second branch assumes, in contrast, that banks have more information than borrowers. Such expertise can take several forms. Investors, such as banks, have a long experience and can better evaluate the quality of projects than individuals who form biased estimates of their prospects (de Meza and Southeys (1996), Manove and Padilla (1999) or Landier and Thesmar (2009)). Garnaize (2007) assumes that outside investors have better expertise in project evaluation than entrepreneurs. He shows that parties may restrict to debt and junior equity (call-options) without loss of efficiency.
Manove, Padilla, and Pagano (2001) assume that banks have a better screening technology that helps sorting out bad projects, which an entrepreneur cannot do and thus, help mitigating private and social costs. They show that high collateral may weaken banks' incentives to screen. Admati and Pfeiffer (1994) consider a stage game where the entrepreneur is better informed than outside investors. They show how outside investors can mitigate the incentives of the entrepreneur to continue projects even when it would be optimal to abandon them. Closer to our paper are Bond, Musto, and Yilmaz (2005) and Inderst (2008). They both consider a model of consumer credit where the lender has superior information. The main difference with our approach is that they both consider a pure signaling model, particularly suitable when considering household financing, where the contract offered by the principal only has effect on the household's decision to accept the offer. That is, they propose a model of unilateral asymmetric information. Both papers have in common that the combination of imperfect competition and informational advantage can give rise to abusive lending. They however differ in the way they model competition and, consequently, come up with different conclusions. Whereas Bond, Musto, and Yilmaz (2005) model competition as an increase of the number of firms with identical information, Inderst (2008) assumes that only the incumbent has superior information. Contrary to Bond, Musto, and Yilmaz (2005), Inderst (2008) shows that competition does not necessarily modify the conclusion obtained under monopoly. We differ with these contributions by considering a bilateral asymmetric information framework and by extending the set of feasible contracts. Instead of considering the effect of competition we analyze to what extent, by not constraining the space of contracts, we can still generate predatory financing by the informed party.

The organization of the paper is as follows. The model and assumptions are presented in Section 2. In Section 3 we consider a pure moral hazard framework and the socially optimal contract as two benchmarks. In Section 4 we obtain the equilibrium for a case where the principal is constrained to offer debt contracts. The contracting problem is an example of an informed principal problem and we adopt a signaling game framework along the lines of Spence (1973). We show that the equilibrium that gives the highest profit to the investor is a pooling equilibrium. We also show that in that pooling equilibrium there may be too much or too little lending compared with the socially optimal level. Moreover, lending is too aggressive when there is overlending. We show that the overlending result can be overcome with optimal interest rate regulation. In Section 5 we show that the main results extend to a case in which the principal has access to more general contracting environments under some mild conditions. We finally discuss our results and propose relevant applications in Section 6.
2 The model

An entrepreneur with no wealth wants to invest $I$ on a risky project that yields $R$ in case of success and $0$ in case of failure. To do so, he searches for an investor to finance the project. We assume that all the cash generated by the project can be pledged for repayment. All parties are assumed to be risk neutral and the interest rate is normalized to $0$. There are only two verifiable final outcomes: success and failure. When offering a contract, the investor can then play with the transfers offered to the entrepreneur in each state. A complete contract in this context consists of a transfer in case of success and a transfer in case of failure. Our approach is then very much of a mechanism design type. However, the transfers to the agent also have a simple financial contract interpretation.

Indeed, assume that the financier only asks for reimbursement $X$ in case of success. This can be equally thought of as being an equity stake in the project or a debt contract. In contrast, whenever the agent receives positive transfers in both states, this can be interpreted as a bundle of cash and equity.

We consider a setting where the financier is a monopolist in the market and makes a take-it-or-leave-it financing offer to the entrepreneur. Importantly, the financier is considered to be experienced and, consequently, as having access to some private information about the profitability of the project. To formalize such informational advantage, we assume that the probability of success for a given project is $p(\theta, e)$, where $\theta \in [0,1]$ can be thought of as a project-specific technological/market parameter and $e \in [0,1]$ is the effort the entrepreneur has to bring to the project. The cost of effort is given by the function $g(e) = \frac{1}{2}e^2$ and we assume that $c > R > I$. The first condition is used to avoid corner solutions in which effort is equal to $1$ and the second one guarantees that at least some projects are worth being financed. In the core of the paper, we assume that $\theta$ and $e$ are complementary, in the sense that the marginal productivity of effort increases with $\theta$, i.e., $p(\theta, e) = \theta e$ and we later discuss the effects of relaxing such condition by assuming that the marginal productivity of effort is independent of $\theta$. Importantly, $\theta$ is only observable by the financier, while $e$ is only observed by the entrepreneur. Neither $\theta$ nor $e$ are verifiable. Still, it is common knowledge that $\theta$ is distributed according to a pdf $F(\cdot)$ with full support on $[0,1]$.

Given such informed principal type of framework, the financier can use her contractual offer to signal her private information. Any information transmitted will, in return, affect the effort level chosen by the entrepreneur. Our aim is to investigate the ability/cost of the principal to fully signal her private information and the extent to which she wants to do so, to identify the projects that will receive financing and, finally, to discuss both efficiency and potential abusive practices regarding the
financing policy of the principal. We assume that the entrepreneur is protected by limited liability. Since he has no initial wealth, any financing offer is constrained by the fact that the entrepreneur cannot put money in the project. That is, transfers from the investor to the entrepreneur are restricted to be positive. Similarly, the entrepreneur can pay back only in case of success and up to $R$. We first consider a setting where the principal is constrained to offer a debt contract where, by definition, the agent ends up with no return in case of failure of the project. In a second step, we consider an unconstrained contracting framework where the principal might decide to commit to offer a positive transfer $Y$ to the agent even in case of failure.

To summarize, we consider a game with the following timing.

1. Nature draws the value of $\theta$ from the distribution $F(\cdot)$, that is privately observed by the financier.

2. The financier decides whether to finance the project or not. If the project fails to get financed, the game ends and both parties get 0. If the project is financed, the financier asks for a certain repayment in case of success, $X \leq R$ and may also offer a transfer $Y \geq 0$ in case of failure.\(^3\)

3. The entrepreneur accepts or rejects the offer. If he rejects, the game ends and both parties get 0. If he accepts, he chooses a level of effort, $e$.

4. Nature decides whether the project succeeds or fails according to the distribution $p(\theta, e) = \theta e$.

5. Contracts are enforced and payoffs are realized.

Call $\alpha \in \{0, 1\}$ the financing decision ($\alpha = 0$ means no financing).\(^4\) Then, if the entrepreneur accepts the offer, the investor’s utility function is

$$\pi(\theta, X, e) = \alpha \left[ p(\theta, e)X - (1 - p(\theta, e))Y - I \right],$$

and the entrepreneur’s utility function is

$$u(\theta, X, e) = \alpha \left[ p(\theta, e)(R - X) + (1 - p(\theta, e))Y - \frac{c^2}{2} \right].$$

Note that the entrepreneur always accepts the contract offer, since he can guarantee a 0 utility by choosing $e = 0$.

\(^3\)Limited liability implies that we can restrict to reimbursements $X \leq R$ and transfers $Y \geq 0$.

\(^4\)In principle, we could assume that the financier could choose any $\alpha \in [0, 1]$. However, since the profit function is increasing in $\theta$ and $\theta$ is a continuous variable, mixed strategies in financing would be used in equilibrium with probability 0.
3 Two Benchmarks

Before considering the different equilibria of the game, let us discuss, as benchmarks, two different settings. We first analyze the complete information game and, in a second step, the choice of a benevolent social planner.

3.1 Complete information with moral hazard

Suppose first that \( \theta \) is common knowledge. Since offering a transfer in case of failure cannot serve incentive purposes, it is straightforward that under complete information, any optimal contract specifies \( Y(\theta) \equiv 0 \). We then have a simple moral hazard problem in which the financier has to induce the entrepreneur to undertake the right level of effort. Given \( \alpha = 1 \) and \( X \), the moral hazard constraint is

\[
e(X, 1, \theta) = \arg \max_{\hat{e} \in [0,1]} \{ p(\theta, \hat{e})(R-X) - g(\hat{e}) \} = \arg \max_{\hat{e} \in [0,1]} \left\{ \theta \hat{e}(R-X) - \frac{c}{2}(\hat{e})^2 \right\}.
\]

This is a concave problem so the first order conditions are necessary and sufficient. This gives:

\[
e(X, 1, \theta) = \max \left\{ 0, \theta \frac{(R-X)}{c} \right\}.
\]

The financier then solves the following problem:

\[
\max_{\alpha, e, X} \quad \alpha(\theta e X - I),
\]

s.t. \( e = e(X, 1, \theta) \).

We use (1) to replace for \( e \) and then obtain that the optimal reimbursement is

\[
X^F(\theta) = \frac{R}{2}.
\]

Plugging back this expression in (1), we get the equilibrium level of effort:

\[
e^F(\theta) = \frac{\theta R}{2c}.
\]

Finally, the financier will decide to finance only those projects that give her a positive net expected profit:

\[
\pi^F(\theta) = \theta e^F(\theta) X^F(\theta) - I \geq 0
\]

Thus, there is a threshold, \( \theta^F \) such that the project gets financed (\( \alpha^F(\theta) = 1 \)) if and only if \( \theta \geq \theta^F \) which, using (2) and (3), is given by

[7]
\[ \theta^F = \frac{2\sqrt{cI}}{R}. \]  

(4)

In what follows, we assume that \( R > 2\sqrt{cI} \), so that some projects are actually being financed. An interesting feature of the complete information case is that the sharing rule \( \frac{X}{R} \) is independent of \( \theta \). In addition, the threshold \( \theta^F \) is increasing with costs parameters (\( c \) and \( I \)) and decreasing with the value \( R \) generated by the project.

### 3.2 Social Planner

Still assuming that \( \theta \) is common knowledge, as a second benchmark we study the choice of a benevolent social planner. Being fully informed of the value of \( \theta \), his objective is to choose the level of effort that maximizes the NPV (net of the cost of effort) and finance all projects with a positive NPV. That is, he chooses the level of effort to maximize total surplus which is equivalent to

\[
(\alpha^W(\theta), e^W(\theta)) = \arg \max_{\tilde{\alpha}, \tilde{e}} \left\{ \tilde{\alpha} \left( \theta eR - \frac{c}{2}\tilde{e}^2 \right) \right\},
\]

or,

\[
e^W(\theta) = \frac{\theta R}{c}.
\]  

(5)

The social planner finances all projects such that

\[
\theta e^W(\theta)R \geq \frac{c}{2}[e^W(\theta)]^2 + I.
\]

Hence, there exists a threshold \( \theta^W \) such that the project gets financed (\( \alpha^W(\theta) = 1 \)) if and only if \( \theta \geq \theta^W \). Using (5), we get that

\[
\theta^W = \frac{\sqrt{2cI}}{R}.
\]  

(6)

Our first result is just a restatement of the inefficiencies given to the moral hazard constraints.

**Proposition 1** Under full information there is too little financing, i.e., \( \theta^W < \theta^F \).

**Proof.** It directly stems from the comparison of the thresholds given in conditions (4) and (6).  

One can be tempted to associate such restriction on the financing of projects to the standard monopoly restriction. This is not the case here since the monopolist can discriminate among types by making a contracting proposal contingent on \( \theta \). The intuition for such a result follows from
Jensen and Meckling (1976). Under full information, the investor has to trade off the magnitude of incentives he provides to the agent (through $R - X$) and his return $X$. Both forces go in opposite direction. In contrast, a social planner is not concerned about the way the surplus is divided but rather solely on total surplus. Therefore, comparing the effort levels under both benchmarks, we get that (weakly) less effort is exerted under full information (as it is costly for the investor to induce effort) compared to the socially optimal level of effort. This implies a lower expected profitability of any given project under full information and therefore explains the lower financing rate under full information.

A crucial assumption is that the entrepreneur is protected by limited liability. This naturally restricts the space of contracts by excluding, say, fixed transfers to the principal since the entrepreneur is unable to pay any positive amount in case of failure. Without limited liability the moral hazard problem disappears and the investor would be able to implement the socially optimal financing level.

4 Debt Contracts with an Informed Principal

In this section, we restrict the set of feasible contracts by assuming that the investor cannot offer transfers to the entrepreneur in case of failure, i.e., $Y(\theta) \equiv 0$. Thus, we can interpret any feasible contract as a debt contract in which the investor decides the interest rate of the loan defined by $X/I - 1$. From an ex-ante perspective, the only information that the entrepreneur is aware of is that the probability of success of his project depends on a technological parameter $\theta$ that is distributed according to the cumulative distribution function $F(\theta)$ over $\theta \in [0, 1]$. In a context where the financier is endowed with some private information, his contractual proposal to the entrepreneur could allow the latter to make his effort choice under a better information structure.

4.1 The Equilibrium Concept

We look for a Perfect Bayesian Equilibrium (PBE) which consists of a (mixed) strategy $(\alpha^*(\theta), q^*(X, \theta))$ for the principal, a strategy $e^*(X, 1)$ and conditional beliefs $\beta(\theta|X, \alpha)$ for the agent such that

1. Financing occurs with probability 1 if the investors’s profit is positive and 0 if it is negative.

That is,

$$
\alpha^*(\theta) = \begin{cases} 
1 & \text{if } \max_{X \in [0, R]} \left\{ p(\theta, e^*(\tilde{X}, 1))\tilde{X} - I \right\} \geq 0 \\
0 & \text{otherwise.}
\end{cases}
$$
2. For any $\theta$ such that $\alpha^*(\theta) = 1$, all reimbursement levels observed with positive probability in equilibrium maximize the principal’s expected payoff given the entrepreneur’s equilibrium strategy and beliefs. That is, for all $X$ such that $q^*(X, \theta) > 0$ we have

$$X \in \arg \max_{\tilde{X} \in [0,R]} p(\theta, e^*(\tilde{X}, 1)) \tilde{X}$$

Another way to write such condition is that for any pair $(X, \tilde{X}) \in [0,R]^2$ we have

$$p(\theta, e^*(X, 1))X \geq p(\theta, e^*(\tilde{X}, 1))\tilde{X}, \quad (7)$$

if $q^*(X, \theta) > 0$.

3. The equilibrium effort $e^*(X, 1)$ maximizes the agent’s expected payoff given his beliefs about $\theta$. That is

$$e^*(X, 1) \equiv e^*(X, \beta(\theta|X, 1)) \in \arg \max_{\tilde{e} \in [0,1]} \int [p(\theta, \tilde{e})(R - X) - g(\tilde{e})]d\beta(\theta|X, 1).$$

This is a strictly concave problem so the first order conditions are necessary and sufficient. Moreover, for given beliefs, the solution is unique, so mixed strategies are never used by the agent. This implies that $e^*(X, 1)$ is implicitly defined by:

$$e e^*(X, 1) = \int \frac{\partial p}{\partial \tilde{e}}(\theta, e^*(X, 1))(R - X)d\beta(\theta|X, 1). \quad (8)$$

4. The agent’s posterior beliefs conditional on observing an equilibrium reimbursement level, $\beta(\theta|X, 1)$, satisfy Bayes’ rule whenever possible.

That is, we assume that the only means for the financier to signal the information she has is through the repayment asked to the entrepreneur. In particular, she might like to ask different types for different payments and, consequently, generate different effort levels.

### 4.2 The Equilibrium Contract

We now turn to the derivation of the equilibrium contract. The following proposition is a first step in this direction.

**Proposition 2** There is a continuum of separating, pooling and semi-separating Perfect Bayesian Equilibria of this game. All those equilibria may be supported by out-of-equilibrium beliefs that assign probability 1 to type 0 if a deviation is observed.
**Proof.** We prove this result by proving the following three lemmas in the Appendix.

**Lemma 1** There is a continuum of separating equilibria in which the investor finances all types above some threshold \( \theta^S \in [\theta^S, 1] \), \( \theta^S = \theta^F \) with

\[
X^S(\theta) = \frac{R}{2} \left( 1 \pm \sqrt{1 - \frac{4Ic}{\theta \theta^S R^2}} \right), \quad (9)
\]

\[
e^S(\theta) = \frac{\theta R}{2c} \left( 1 \mp \sqrt{1 - \frac{4Ic}{\theta \theta^S R^2}} \right), \quad (10)
\]

The separating equilibrium that gives the highest profit to the investor has \( \theta^S = \theta^S \).

**Lemma 2** There is a continuum of semi-separating equilibria with pooling on some subset \( \hat{\Theta} \subset [\theta^S, 1] \) and separation on \( \hat{\Theta} = [\theta^S, 1]/\hat{\Theta} \), in which 1) the investor finances all types above some threshold \( \theta^SP \) for \( \theta^S \in [\theta^S, 1] \), with \( \theta^S \) defined such that \( \theta^S \min \{ \inf \hat{\Theta}, \mathbb{E}(\theta | \theta \in \hat{\Theta}) \} = \frac{4Ic}{R^2} \), 2) reimbursement and effort are given by

\[
X^SP(\theta) = \begin{cases} \frac{R}{2} \left( 1 \pm \sqrt{1 - \frac{4Ic}{\mathbb{E}(\theta | \theta \in \hat{\Theta}) \theta^SP R^2}} \right) & \text{if } \theta \in \hat{\Theta}, \\ \frac{R}{2} \left( 1 \mp \sqrt{1 - \frac{4Ic}{\mathbb{E}(\theta | \theta \in \hat{\Theta}) \theta^SP R^2}} \right) & \text{if } \theta \in \hat{\Theta}, \end{cases}
\]

\[
e^SP(\theta) = \begin{cases} \mathbb{E}(\theta | \theta \in \hat{\Theta}) \frac{R}{2c} \left( 1 \mp \sqrt{1 - \frac{4Ic}{\mathbb{E}(\theta | \theta \in \hat{\Theta}) \theta^SP R^2}} \right) & \text{if } \theta \in \hat{\Theta}, \\ \frac{\theta R}{2c} \left( 1 \pm \sqrt{1 - \frac{4Ic}{\theta \theta^S R^2}} \right) & \text{if } \theta \in \hat{\Theta}. \end{cases}
\]

The semi-separating equilibrium that gives the highest profit to the investor has \( \theta^SP = \theta^SP \).

**Lemma 3** There is a continuum of pooling equilibria in which 1) the investor finances all types above some threshold \( \theta^P \in [\theta^P, 1] \) for \( \theta^P \) defined such that \( \theta^P \mathbb{E}(\theta | \theta \geq \theta^P) = \frac{4Ic}{R^2} \), 2) reimbursement and effort are given by

\[
X^P = \frac{R}{2} \left( 1 \mp \sqrt{1 - \frac{4Ic}{\mathbb{E}(\theta | \theta \geq \theta^P) \theta^P R^2}} \right), \quad (11)
\]

\[
e^P = \mathbb{E}(\theta | \theta \geq \theta^P) \frac{R}{2c} \left( 1 \mp \sqrt{1 - \frac{4Ic}{\mathbb{E}(\theta | \theta \geq \theta^P) \theta^P R^2}} \right), \quad (12)
\]

The pooling equilibrium that gives the highest profit to the investor has \( \theta^P = \theta^P \).

The derivation and the characteristics of the equilibria are shown in Appendix A. Signaling games are usually characterized by a multiplicity of equilibria and this game is not an exception. Almost any contract can be an equilibrium arrangement under the condition that it is incentive
compatible. However, we will focus our analysis on the equilibrium that gives the highest profit to the investor.\footnote{It turns out that, in this context of debt contracts, this equilibrium is the only one that survives Grossman and Perry (1986)’s refinement. A formal proof is provided in Appendix B.} According to Proposition 2, there are three classes of equilibria: separating, pooling and semiseparating. Within each class, we can select one equilibrium that dominates the others from the investor’s point of view. That is, we can identify a “best separating equilibrium”, a “best pooling equilibrium” and a “best semiseparating equilibrium”. The next proposition shows that the best pooling equilibrium dominates the other two, for any $\theta$.

**Proposition 3** For any $\theta \in [0, 1]$, the payoff of the principal is higher in the best pooling equilibrium than in any other equilibrium, pooling or not.

**Proof.** Consider the best of the 3 types of equilibria (separating, semi-separating and pooling). Call $\pi^S(\theta)$ the investor’s payoff in the best separating equilibrium. Similarly, define $\pi^{SP}(\theta)$ and $\pi^P(\theta)$. According to the proof of Proposition 2, for a type $\tilde{\theta}$ who gets financed the investor’s profits under each type are

$$
\pi^P(\theta) = \theta \mathbb{E}(\theta | \theta \geq \theta^P) \frac{R^2}{4c} - I, \\
\pi^{SP}(\theta) = \theta \mathbb{E}(\theta^S | \theta \geq \theta^P) \frac{R^2}{4c} - I, \\
\pi^S(\theta) = \theta \mathbb{E}(\theta) \frac{R^2}{4c} - I,
$$

all increasing in $\theta$ where

$$\tilde{\theta} \equiv \min\{\inf \bar{\Theta}; \mathbb{E}(\theta | \theta \in \bar{\Theta})\}.$$

Moreover, by definition of $\theta^P$, $\theta^{SP}$ and $\theta^S$ we know that

$$\pi^P(\theta^P) = \pi^{SP}(\theta^{SP}) = \pi^S(\theta^S) = 0.$$

This determines that

$$\theta^P \mathbb{E}(\theta | \theta \geq \theta^P) = \theta^{SP} \mathbb{E}(\theta^S) = (\theta^S)^2 = \frac{4Ic}{R^2}. \quad (13)$$

Since $\theta^P \leq \mathbb{E}(\theta | \theta \geq \theta^P)$, (13) implies that $\mathbb{E}(\theta | \theta \geq \theta^P) \geq \theta^S \geq \theta^P$ and $\pi^P(\theta) \geq \pi^S(\theta)$ for any $\theta$. Suppose now that $\theta^P > \theta^{SP}$. Then, (13) implies that

$$\theta^P \leq \mathbb{E}(\theta | \theta \geq \theta^P) < \tilde{\theta}. \quad (14)$$

Moreover, by definition we know that

$$\bar{\theta} \leq \mathbb{E}(\theta | \theta \in \bar{\Theta}), \quad (15)$$

$$\bar{\theta} \leq \inf \bar{\Theta}. \quad (16)$$

Combining (14) and (15) we get that

$$\int_{\theta^P}^{\tilde{\theta}} (\theta - \bar{\theta})d\mathbb{F}(\theta) < 0 < \int_{\theta \in \bar{\Theta}} (\theta - \bar{\theta})d\mathbb{F}(\theta). \quad (17)$$
Since \( \Theta = \Theta \cup \Theta - [\theta^P, \theta^P] \), (17) becomes
\[
\int_{\theta \in \Theta} (\theta - \tilde{\theta})dF(\theta) + \int_{\theta \in \Theta} (\tilde{\theta} - \theta)dF(\theta) < 0,
\]
which is a contradiction since (16) implies that the first term is positive and \( \tilde{\theta} > \theta^P > \theta^S \) implies that the second term is positive too. Hence, we have proved that \( \theta^P \leq \theta^S \), which implies that \( \theta < \mathbb{E}(\theta|\theta \geq \theta^P) \) and \( \pi^P(\theta) \geq \pi^S(\theta) \) for any \( \theta \).

The equilibrium that maximizes the investor’s profits (the best equilibrium) is the best pooling equilibrium. In this equilibrium, the investor finances all types above \( \theta^P (< \theta^F) \) with
\[
X^P = \frac{R}{2},
\]
\[
e^P = \mathbb{E}(\theta|\theta \geq \theta^P)\frac{R}{2c}.
\]
This implies that in the best equilibrium the investor is able to implement the full information reimbursement, at the cost of a distortion on the level of effort of the entrepreneur. Suppose that there are many periods and that the entrepreneur learns the value of \( \theta \) after the first period, then the reimbursement level would not change, but effort would adjust. The intuition for the fact that \( \theta^P < \theta^F \) stems precisely from the fact that, given that the pay-back \( X^P \) imposed under the pooling equilibrium corresponds to \( X^F \) (full information), the break-even condition entails that
\[
\theta^P \mathbb{E}(\theta|\theta \geq \theta^P) = (\theta^F)^2 \text{ which implies } \theta^P < \theta^F < \mathbb{E}(\theta|\theta \geq \theta^P).
\]
The entrepreneur then exerts too much effort if his type is low (lower than \( \mathbb{E}(\theta|\theta \geq \theta^P) \)) and too little effort if his type is high. In other words, with the same reimbursement policy than under full information, types close to the threshold type make more effort than under full information. This implies that the investor’s profit on those types is higher in the pooling equilibrium. This and the fact that \( e(\theta)X(\theta) \) is constant in any equilibrium of the signaling game explain why the pooling equilibrium dominates the separating and semi-separating equilibria.

Let us now explore the potential inefficiencies in the behavior of the principal and characterize the properties of the best equilibrium in terms of the set of types who receive financing. This is the purpose of the following two propositions.

**Proposition 4** There are two potential inefficiencies in equilibrium (i) If \( \theta^P < \frac{\mathbb{E}(\theta|\theta \geq \theta^P)}{2} \), then \( \theta^P < \theta^W \) and, therefore, the investor over-lends to the entrepreneur. (ii) If \( \theta^P > \frac{\mathbb{E}(\theta|\theta \geq \theta^P)}{2} \), then \( \theta^P > \theta^W \) and the equilibrium level of financing is sub-optimal but the magnitude of the inefficiency is lower than under complete information.
**Proof.** $\theta^W$ and $\theta^P$ are defined by

$$\theta^W = \frac{\sqrt{2cI}}{R},$$

$$\theta^P \mathbb{E}(\theta \geq \theta^P) = \frac{4cI}{R^2}.$$ 

Thus,

$$\left(\frac{\theta^W}{\theta^P}\right)^2 = \frac{1}{2} \frac{\mathbb{E}(\theta \geq \theta^P)}{\theta^P}.$$ 

Hence, $\theta^W > \theta^P$ if and only if $\mathbb{E}(\theta \geq \theta^P) > 2\theta^P$.  

One interesting property of the pooling equilibrium is that it may generate inefficient lending ($\theta^P < \theta^W$). This never happens under full information (see Proposition 1). To give an intuition for when this could happen, fix a distribution of $\theta$ and define the set

$$P = \{x \in [0, 1/2] : 2x < \mathbb{E}(\theta \geq x)\}.$$ 

Then inefficient lending occurs whenever $\theta^P \in P$. Note that $P \neq \emptyset$ because $0 \in P$. Moreover, $P$ is larger when the distribution is more skewed to the right. This implies that inefficient lending is more likely to occur if the distribution puts a high weight on high types. On the other hand, for a given set $P$, the chances that $\theta^P$ fall in $P$ decrease with $c$ and $I$ and increase with $R$. To see an example, suppose that the distribution of $\theta$ is uniform on $[0, 1]$. Then, $P = [0, 1/3)$ and the condition for inefficient lending is met if $\theta^P < 1/3$. For instance, this happens if $R = 10$, $I = 1/6$, $c = 12$. In this case, $\theta^P \approx 0.14$, $\theta^W \approx 0.2$ and $\theta^F \approx 0.28$ and lending is inefficient if $\theta \in [0.14, 0.2]$. If instead $I = 1$, we obtain $\theta^P \approx 0.6$, $\theta^W \approx 0.49$ and $\theta^F \approx 0.69$ and lending is always efficient when it occurs.

Now, we define abusive lending as a setting where the expected utility of the entrepreneur, conditional on his effort and the investor’s information is negative. That is, the entrepreneur is worse off once developing his project than having refused to receive financing. In our context, we get the following result.

**Proposition 5** In the best equilibrium the set of types for which there is abusive lending is non-empty if and only if there is over-lending.

**Proof.** The entrepreneur’s expected utility when he gets financing and his type turns out to be $\theta$ is

$$\theta e^P (R - X^P) - \frac{1}{2} c(e^P)^2 = \mathbb{E}(\theta \theta \geq \theta^P) \frac{R^2}{4c} \left(\theta - \frac{\mathbb{E}(\theta \theta \geq \theta^P)}{2}\right).$$ 

Thus, the entrepreneur’s ex post utility is negative if and only if $\theta < \frac{\mathbb{E}(\theta \theta \geq \theta^P)}{2}$. The set of types that get negative expected utility conditional on the investor’s information is

$$\{x \in [\theta^P, 1] : 2x < \mathbb{E}(\theta \theta \geq \theta^P)\} = [0, \mathbb{E}(\theta \theta \geq \theta^P)/2) \cap [\theta^P, 1],$$

14
which is non-empty if and only if \(2\theta^P < \mathbb{E}(\theta|\theta \geq \theta^P)\).

Propositions 4 and 5 highlight two interesting characteristics of the financing offer. First, there can be over-lending by the investor compared to the social planner’s benchmark. Moreover, in those cases in which there is over-lending, an entrepreneur with a low enough \(\theta\) would have been better off if he did not accept the financing offer in the first place. This feature can easily be seen from considering a type \(\theta \in \left[\theta^P, \frac{\mathbb{E}(\theta|\theta \geq \theta^P)}{2}\right]\) whose utility is given by
\[
\frac{P^2}{\pi} \mathbb{E}(\theta|\theta \geq \theta^P) \left(\theta - \frac{\mathbb{E}(\theta|\theta \geq \theta^P)}{2}\right) < 0.
\]
This negative expected utility stems from the fact that his effort is too high compared to the effort he would have exerted if he had known his type. When there is abusive lending, it is always against low types. However, abusive lending occurs both against all types for which lending is inefficient (\(\theta < \theta^W\)) and also against some types for which lending is actually efficient (\(\theta \geq \theta^W\)) provided that their type is sufficiently close to \(\theta^W\).

### 4.3 Interest rate regulation

Previous results suggest that it could be welfare improving to regulate this market by imposing a cap to the interest rate that the investor can charge. The reason is that a lower interest rate reduces the profits of the investor on any type and, in particular, on the lowest types who are making too much effort. Indeed, suppose that a planner were to choose the interest rate in order to maximize social welfare, taking into account that following this regulation the market coordinates, as before, on the corresponding pooling equilibrium. In this model, fixing the interest rate is equivalent to fixing \(X\). Following the proof of Lemma 3, we know that for a given \(X\) the entrepreneur’s effort is
\[
e(\bar{X}) = \mathbb{E}(\theta|\theta \geq \bar{\theta}) \frac{R - \bar{X}}{c},
\]
where \(\bar{\theta}\) is the lowest type who gets financed and is defined by the zero profit condition:
\[
\bar{\theta}\mathbb{E}(\theta|\theta \geq \bar{\theta}) \frac{R - \bar{X}}{c} \bar{X} = I.
\]
Call
\[
W(\bar{\theta}, \bar{X}) = \int_{\bar{\theta}}^{1} \left[\theta\mathbb{E}(\theta|\theta \geq \bar{\theta}) \frac{R - \bar{X}}{c} R - \frac{1}{2} \mathbb{E}(\theta|\theta \geq \bar{\theta})^2 \frac{(R - \bar{X})^2}{c} \right] dF(\theta)
\]
\[
= (1 - F(\bar{\theta})) \left[\frac{\mathbb{E}(\theta|\theta \geq \bar{\theta})^2}{c} \frac{R - \bar{X}}{c} R - \frac{1}{2} \frac{\mathbb{E}(\theta|\theta \geq \bar{\theta})^2}{c} \frac{(R - \bar{X})^2}{c} - I \right],
\]
\[\text{6}\text{The “if and only if” of Proposition 5 is particular to our specifications. However, it is general that if there is over-lending then the set of types whose ex post utility is negative conditional on the principal’s information is non-empty.}\]
the expected welfare under regulation contract \((\bar{\theta}, \bar{X})\). Thus, the planner’s problem writes

\[
\max_{\bar{\theta}, \bar{X}} W(\bar{\theta}, \bar{X}),
\]

subject to

\[
\bar{\theta}\mathbb{E}(\theta|\bar{\theta}) \geq \bar{\theta} \frac{R - \bar{X}}{c} \bar{X} = I.
\]

**Proposition 6** At the optimal regulation policy some projects get financed \((\bar{\theta} < 1)\), the interest rate is lower than at the best unregulated pooling equilibrium \((\bar{X} < R/2)\) and there is never over-lending \((\bar{\theta} > \theta^W)\).

**Proof.** To simplify notation define

\[
h(\bar{\theta}) = \mathbb{E}(\theta|\bar{\theta} \geq \bar{\theta}).
\]

Hence,

\[
h'(\bar{\theta}) = \frac{f(\bar{\theta})}{1 - F(\bar{\theta})}.
\]

a) Suppose first that \(\bar{\theta} = 1\). Then, for any \(\bar{X}\), \(W(1, \bar{X}) = 0\). Now consider

\[
\bar{X} = \frac{R}{2} \text{ and } \bar{\theta} = \theta^P < 1: \theta^P h(\theta^P) = \frac{4Ic}{R^2}.
\]

Then,

\[
W\left(\theta^P, \frac{R}{2}\right) = (1 - F(\theta^P)) \left(h(\theta^P)^2 \frac{R^2}{2c} - \frac{1}{2} h(\theta^P)^2 \frac{R^2}{4c} - I\right).
\]

Using the fact that

\[
\theta^P h(\theta^P) = \frac{4Ic}{R^2},
\]

\[
W\left(\theta^P, \frac{R}{2}\right) = (1 - F(\theta^P)) \left(h(\theta^P)^2 \frac{R^2}{2c} - h(\theta^P)^2 \frac{R^2}{8c} - \theta^P h(\theta^P) \frac{R^2}{4c}\right),
\]

\[
= (1 - F(\theta^P)) h(\theta^P) \frac{R^2}{8c} (3h(\theta^P) - 2\theta^P) > 0,
\]

because \(h(\theta^P) > \theta^P\). Thus, it is always optimal to finance some types.

b) From the constraint, we have that

\[
\bar{\theta} h(\bar{\theta}) \frac{R - \bar{X}}{c} \bar{X} = I.
\]

By definition

\[
\theta^P h(\theta^P) \frac{R^2}{4c} = I
\]

and

\[
\frac{R - \bar{X}}{c} \bar{X} \leq \frac{R^2}{4c}
\]

for any \(X\). Hence, it must be that \(\bar{\theta} \geq \theta^P\). So consider \(\bar{\theta} \geq \theta^P\). Then, there are two values of \(\bar{X}\) that satisfy the constraint: \(X_1 = \frac{R}{2} - \Delta(\bar{\theta})\) and \(X_2 = \frac{R}{2} + \Delta(\bar{\theta})\), with

\[
\Delta(\bar{\theta}) = \frac{1}{2} \sqrt{R^2 - \frac{4Ic}{\bar{\theta} h(\bar{\theta})}} \geq 0.
\]
We now compute
\[ W(\hat{\theta},X_1) - W(\hat{\theta},X_2) = (1 - F(\hat{\theta}))h(\hat{\theta})^2 R/c \Delta(\hat{\theta}) \geq 0. \]

Therefore, for any feasible \( \hat{\theta} \) social welfare is larger when \( X \leq R/2 \).

In points a) and b) we have shown that there are no corner solutions. Hence, the solution must be interior and satisfy the first order conditions. The lagrangean of the planner’s problem is
\[
L = (1 - F(\hat{\theta})) \left[ h(\hat{\theta})^2 R - \bar{X} \right] - \frac{1}{2} h(\hat{\theta})^2 \left( \frac{R - \bar{X}}{c} \right)^2 - I + \lambda \left( \theta h(\hat{\theta}) \frac{R - \bar{X}}{c} - \bar{X} - I \right).
\]

From the first order conditions, we obtain that:

\[
0 = \frac{\partial L}{\partial \theta} = -f(\hat{\theta}) \left[ h(\hat{\theta})^2 \frac{R - \bar{X}}{c} - \frac{1}{2} h(\hat{\theta})^2 \left( \frac{R - \bar{X}}{c} \right)^2 - I \right] + (1 - F(\hat{\theta})) \left[ 2h(\hat{\theta}) \frac{R - \bar{X}}{c} R\hat{\theta}'(\hat{\theta}) - h(\hat{\theta}) \left( \frac{R - \bar{X}}{c} \right)^2 \hat{\theta}(\hat{\theta}) \right]
+ \lambda \left( \frac{R - \bar{X}}{c} \bar{X} \left[ h(\hat{\theta}) + \hat{\theta}(\hat{\theta}) \right] \right)
\]

\[ 0 = \frac{\partial L}{\partial X} = (1 - F(\hat{\theta})) \left[ h(\hat{\theta})^2 \frac{R - \bar{X}}{c} + h(\hat{\theta})^2 \frac{R - \bar{X}}{c} \right] + \lambda \theta h(\hat{\theta}) \frac{R - 2\bar{X}}{c}
\]

\[ 0 = \frac{\partial L}{\partial \lambda} = \theta h(\hat{\theta}) \frac{R - \bar{X}}{c} \bar{X} - I. \]

First note that, according to (20), \( \bar{X} > 0 \), because \( I > 0 \).

Now, simplifying (19) we obtain
\[ \lambda \theta (R - 2\bar{X}) = (1 - F(\hat{\theta}))h(\hat{\theta}) \bar{X} > 0. \]

This, together with \( \bar{X} \leq R/2 \), implies \( \bar{X} < R/2 \) and \( \lambda > 0 \). Finally, simplifying (18) and using the constraint we get
\[ \lambda \bar{X} \left( h(\hat{\theta}) + \theta (h(\hat{\theta}) - \hat{\theta}) \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \right) = \frac{1}{2} f(\hat{\theta}) h(\hat{\theta}) \left( R(2\hat{\theta} - h(\hat{\theta})) - h(\hat{\theta}) \bar{X} \right). \]

The left-hand-side is positive, so the right-hand-side must be positive too, implying that
\[ 2\hat{\theta} > h(\hat{\theta}). \]

Now, by definition, we have that
\[ \theta h(\hat{\theta})(R - \bar{X}) \bar{X} = cI = \left( \theta^w \right)^2 \frac{R^2}{2}. \]

Moreover, for any \( \bar{X} \)
\[ (R - \bar{X}) \bar{X} \leq \frac{R^2}{4}. \]

Hence,
\[ \left( \theta^w \right)^2 \frac{R^2}{2} = \theta h(\hat{\theta})(R - \bar{X}) \bar{X} \leq \theta h(\hat{\theta}) \frac{R^2}{4}, \]
or
\[ \left( \theta^w \right)^2 \leq \frac{1}{2} \theta h(\hat{\theta}) < (\hat{\theta})^2, \]

17
where the last inequality comes from (21). Thus $\tilde{\theta} > \theta^W$ and there is never overlending.

The optimal regulation policy entails less financing than the unregulated equilibrium and less financing than the (full information) social optimum: $\tilde{\theta} > \max\{\theta^P, \theta^W\}$. The comparison between $\tilde{\theta}$ and $\theta^F$ is not straightforward. Depending on the values of the parameters $\tilde{\theta}$ could be larger or smaller than $\theta^F$.

4.4 The Importance of Complementarity

In this section, we show that some type of complementarity between the private information of the investor and the decision of the entrepreneur is crucial for our results to hold. By complementarity, we mean that the beliefs of the entrepreneur about the investor’s information have an impact on the entrepreneur’s decision. In the model presented so far, different beliefs about $\theta$ entail different effort decisions by the entrepreneur. Similarly, in Bond, Musto, and Yılmaz (2005), different beliefs may change the borrower’s decision of accepting the loan.

Consider indeed the exact model we described in Section 2 except that there is no complementarity between the investor’s information and the level of effort ($p_{12}(\theta, e) = 0$). To avoid corner solutions, we assume that $p(\theta, e) = \frac{\theta + e}{2}$. The results are in sharp contrast with the ones we highlighted above. Indeed, we can show that the complete information (semi-separating) equilibrium contract is the only equilibrium of the game and, therefore, that signaling is costless to the principal. This is the aim of the following proposition.

**Proposition 7** When $\theta$ and $e$ are perfect substitutes, the complete information contract is the only equilibrium of the game.

A direct implication of Proposition 7 is that signaling occurs at no cost for the investor when the structure of the probability of success entails perfect substitution between $\theta$ and $e$. To be more precise, observe that given the semi-separating structure of the equilibrium, information on $\theta$ is only partially transmitted to the agent. However, for the subset of types for which the equilibrium is pooling, the equilibrium level of effort is invariably null. That is, all the relevant information is transmitted at no cost at equilibrium. Under perfect substitution, $\theta$ has no impact on the marginal productivity of effort. This then entails that awareness of the true value of $\theta$ should not change the effort decision of the agent and, consequently, explains why the effort decisions coincide. Now, since the financier is, in both settings, informed of the true realization of $\theta$ then, clearly, the only equilibrium is to propose the same transfers as in the complete information framework. Now, there is always sub-optimal financing at equilibrium whenever $\theta$ and $e$ are perfect substitutes.
5 Equilibrium with unconstrained contracts

So far, we have assumed that, in case of failure, none of the contracting parties receives a positive transfer. This clearly constrains the contract to what could be considered as a debt contract. Suppose now that the financier has a deep pocket and that he can commit to offer a transfer $Y(\theta) \geq 0$ to the agent in case of failure together with a reimbursement $X(\theta) \leq R$ in case of success of the project. It is straightforward to reinterpret such a contract as providing a fixed transfer $Y(\theta) \geq 0$ independent of the outcome associated to a reimbursement $Z(\theta) = Y(\theta) + X(\theta) \leq R + Y(\theta)$ in case of success.

As we already discussed, offering a transfer in case of failure cannot serve incentive purposes. In contrast, it provides an additional instrument for the principal to signal its information to the agent. In such a context, whenever considering pooling contracts, it is straightforward that the associated transfer in the low state remains null when we consider the best pooling equilibrium. Indeed, with no signaling, a positive transfer in case of failure is only a waste of money. This does not necessarily hold in a separating equilibrium. The following result describes how relaxing the constraint on the contract space modifies the separating equilibrium contractual offer of the principal.

**Proposition 8** The best separating equilibrium from the investor’s perspective is such that the financier finances all types above $\theta^S = \theta^F$ and offers contract

\[
Y(\theta) = \frac{(\theta R)^2}{8c} - \frac{I}{2}, \\
Z(\theta) = \frac{R}{2}.
\]

**Proof.** Suppose there is a separating equilibrium in which the investor makes an offer $(Y(\theta), Z(\theta))$. Observing this offer, the entrepreneur correctly interprets that the true value of the parameter is $\theta$ and chooses the effort level to solve

\[
\max_{e} \left\{ Y(\theta) + \theta e (R - Z(\theta)) - \frac{e^2}{2} \right\}.
\]

Thus,

\[
e(\theta) = \theta \frac{R - Z(\theta)}{c}.
\]

For this to be an equilibrium, it must be that a type $\theta$ financier has no incentives to deviate. That is, for any $\theta \neq \bar{\theta}$, we have

\[
\theta e(\theta) Z(\theta) - Y(\theta) \geq \theta e(\bar{\theta}) Z(\bar{\theta}) - Y(\bar{\theta})
\]

with $e(\theta) = \theta \frac{R - Z(\theta)}{c}$. This is equivalent to checking that the derivative of the profit function $\pi(\theta, \tilde{\theta})$ with respect to $\tilde{\theta}$ is null at $\tilde{\theta} = \theta$. Such a profit function can be written as

\[
\pi(\theta, \tilde{\theta}) = \theta e(\tilde{\theta}) Z(\tilde{\theta}) - Y(\tilde{\theta}) - I.
\]
Using the envelop theorem and the fact that $e(\theta) = \theta \frac{R - Z(\theta)}{c}$, we get
\[
\frac{d\pi}{d\theta}(\theta, \theta) = e(\theta)Z(\theta) = \frac{\theta(R - Z(\theta))}{c} \quad \text{Z(\theta)},
\]
or
\[
\pi(\theta, \theta) = \int_{\theta_0}^{\theta} \frac{u(R - Z(u))}{c} Z(u) \, du. \tag{22}
\]
The best transfers from the investor’s perspective solve the following problem
\[
\max_{\theta \in (\theta_0, \theta_1)} \int_{\theta_0}^{\theta} \frac{u(R - Z(u))}{c} Z(u) \, du \tag{23}
\]
which implies that $Z(\theta) = \frac{R}{c}$ and, consequently $e(\theta) = \theta \frac{R}{2c}$. It remains to determine $Y(\theta)$. This can be obtained by using the expressions of $Z(\cdot)$ and $e(\cdot)$ above and noticing that
\[
\pi(\theta) = \int_{\theta_0}^{\theta} \frac{u(R - Z(u))}{c} Z(u) \, du = \frac{R^2}{8c} \left(\theta^2 - (\theta^S)^2\right)
\]
\[
\pi(\theta) = \theta^2 \frac{R^2}{4c} - Y(\theta) - I.
\]
We then deduce that
\[
Y(\theta) = (\theta^2 + (\theta^S)^2) \frac{R^2}{8c} - I. \tag{24}
\]
$\theta^S$ is the threshold from which the financier accepts to provide financing. It is then defined such that
\[
(\theta^S)^2 \frac{R^2}{4c} - Y(\theta^S) = I.
\]
Finally, in the best separating equilibrium it must be that $Y(\theta^S) = 0$ (there is no need to pay to signal information that is not valuable). Then, we get
\[
\theta^S = \frac{2\sqrt{Tc}}{R} \equiv \theta^P.
\]
Finally, plugging $\theta^S$ in equation (24), we get
\[
Y(\theta) = \frac{(\theta R)^2}{8c} - \frac{I}{2}. \tag{25}
\]

By using two instruments, the principal can achieve two goals. First, through its transfer that is independent of the outcome (but not of \(\theta\)), the principal can continue to signal the information he has on \(\theta\). The larger \(\theta\), the larger the unconditional transfer that is paid by the principal to the agent. This comes at no surprise since signaling a high profile must be somehow costly to the principal. In contrast, by asking for a reimbursement in case of success that is independent of its private information (namely \(R/2\)), the financier can implement the same equilibrium effort as in the complete information setting. With identical effort, the financing policy also coincides with the complete information one. Moreover, the investor’s equilibrium payoff is undoubtedly higher than the one associated to the best separating equilibrium debt contract analyzed in the preceding section.
Following the approach developed in the preceding section, we focus again on the best equilibrium from the investor’s perspective. Denote by \( \forall(\theta) \) the variance of the distribution of \( \theta \) and define \( \alpha(\theta^P) = \frac{2 \varepsilon(\theta|\theta \geq \theta^S) \mathbb{E}(\theta|\theta \geq \theta^P) - \theta^P \mathbb{E}(\theta|\theta \geq \theta^P) - (\mathbb{E}(\theta|\theta \geq \theta^S))^2}{(\mathbb{E}(\theta|\theta \geq \theta^S))^2} \). Note that \( \alpha(\theta^P) \leq 1, \alpha(1) = 0 \) and \( \alpha(0) = 1 \).

**Proposition 9**

a) The best pooling equilibrium maximizes the investor’s profits for any \( \theta \in [0,1] \) if and only if
\[
\mathbb{E}(\theta|\theta \geq \theta^P) \geq \frac{1}{2 - \theta^P},
\]
(26)
b) Otherwise, the best pooling equilibrium maximizes the investor’s profits only from an ex-ante perspective if
\[
\forall(\theta|\theta \geq \theta^S) \leq \alpha(\theta^P) \mathbb{E}(\theta|\theta \geq \theta^S)^2,
\]
(27)

**Proof.** We know that in the pooling equilibrium the investor’s profits are
\[
\pi^P(\theta) = \begin{cases} 
  0 & \text{if } \theta < \theta^P \\
  \theta \mathbb{E}(\theta|\theta \geq \theta^P) \frac{R^2}{4c} - 1 & \text{if } \theta \geq \theta^P.
\end{cases}
\]
According to Proposition 8, in the separating equilibrium, the investor’s profits are
\[
\pi^S(\theta) = \begin{cases} 
  0 & \text{if } \theta < \theta^S \\
  \theta^2 \frac{R^2}{4c} - \frac{l}{2} & \text{if } \theta \geq \theta^S.
\end{cases}
\]
a) Since by definition
\[
\theta^P \mathbb{E}(\theta|\theta \geq \theta^P) = \frac{4c}{R^2} = (\theta^S)^2,
\]
we have that
\[
\pi^P(\theta) - \pi^S(\theta) = \begin{cases} 
  0 & \text{if } \theta \in [0, \theta^P) \\
  \mathbb{E}(\theta|\theta \geq \theta^P) \frac{R^2}{4c} (\theta - \theta^P) & \text{if } \theta \in [\theta^P, \theta^S] \\
  \frac{R^2}{4c} \left[ -\theta^2 + 2\theta \mathbb{E}(\theta|\theta \geq \theta^P) - \theta^P \mathbb{E}(\theta|\theta \geq \theta^P) \right] & \text{if } \theta \in (\theta^S, 1].
\end{cases}
\]

It is easy to see that \( \pi^P(\theta) \geq \pi^S(\theta) \) for \( \theta \leq \theta^S \). Moreover, \( \pi^P(\theta) - \pi^S(\theta) \) is increasing in \( \theta \) if \( \theta \leq \mathbb{E}(\theta|\theta \geq \theta^P) \) and decreasing in \( \theta \) if \( \theta \geq \mathbb{E}(\theta|\theta \geq \theta^P) \). Thus, if \( \pi^P(1) \geq \pi^S(1) \), the pooling equilibrium is the best for the investor for any \( \theta \). \( \pi^P(1) \geq \pi^S(1) \) is equivalent to (26). Hence, under this condition, the investor’s profits are unambiguously higher under the pooling equilibrium.

b) Now, suppose that (26) is not satisfied. This implies that the separating equilibrium gives more profits than the pooling equilibrium when \( \theta \) is high. So from an ex-post perspective it is impossible to compare both

\[\text{\footnote{Unfortunately, under unconstrained contracts no equilibrium survives Grossman-Perry’s refinement. From the best pooling equilibrium, there is a consistent deviation to } (Z', Y') = (R/2, \varepsilon), \text{ for } \varepsilon \text{ small. From the best separating equilibrium, there is a consistent deviation to } (Z', Y') = (R/2 - \varepsilon, 0), \text{ for } \varepsilon \text{ small.}}\]

\[\text{\footnote{Remember that } \theta^S = \sqrt{\theta^P \mathbb{E}(\theta|\theta \geq \theta^P)} \text{ and, therefore, } \alpha(\cdot) \text{ can be defined as a function of } \theta^P \text{ only.}}\]
equilibria. We look now at the ex-ante profits. Again, using the fact that \( \theta P \mathbb{E}(\theta|\theta \geq \theta P) = \frac{4\theta^2}{\pi^2} = (\theta^S)^2 \), we can write the difference in expected profits as

\[
\pi^P - \pi^S = \frac{R^2}{8c} \int_{\theta_S}^1 \left( 2\theta \mathbb{E}(\theta|\theta \geq \theta P) - \theta^2 - \theta P \mathbb{E}(\theta|\theta \geq \theta P) \right) dF(\theta) \\
+ \mathbb{E}(\theta|\theta \geq \theta P) \frac{R^2}{4c} \int_{\theta P}^{\theta^S} (\theta - \theta P) dF(\theta).
\]

The second integral is always positive, so it is enough to ensure that the first integral is positive to guarantee the result. This entails

\[
\int_{\theta_S}^1 \left( 2\theta \mathbb{E}(\theta|\theta \geq \theta P) - \theta^2 - \theta P \mathbb{E}(\theta|\theta \geq \theta P) \right) dF(\theta) \geq 0,
\]

or

\[
\int_{\theta_S}^1 \frac{2\mathbb{E}(\theta|\theta \geq \theta P) \mathbb{E}(\theta|\theta \geq \theta P) - \theta^2 - \theta P \mathbb{E}(\theta|\theta \geq \theta P)}{1 - P(\theta^S)} dF(\theta) \geq 0.
\]

This condition can be rewritten as

\[
\frac{\nabla(\mathbb{E}(\theta|\theta \geq \theta S))}{(\mathbb{E}(\theta|\theta \geq \theta S))^2} \leq \frac{2\mathbb{E}(\theta|\theta \geq \theta^S) \mathbb{E}(\theta|\theta \geq \theta^S) - \theta^2 - \theta P \mathbb{E}(\theta|\theta \geq \theta P) - (\mathbb{E}(\theta|\theta \geq \theta S))^2}{(\mathbb{E}(\theta|\theta \geq \theta S))^2},
\]

which is equivalent to (27).

Condition (26) is necessary and sufficient. Some distributions like uniform, power and logistic do satisfy it for any \( \theta P \in [0, 1] \). Condition (27) is sufficient but not necessary to get the result. Moreover, it is satisfied for a larger set of distributions on \( [0, 1] \) like uniform, Cauchy, power, logistic and beta among others.\(^9\)

When the investor is not constrained to use debt contracts, she can use two instruments for two different purposes: a transfer in case of failure for signaling purposes and a stake in the project in case of success for incentive purposes. This does not alter of course the pooling equilibrium (there is no signaling), but favors the separating equilibrium. Indeed, it is not possible anymore to guarantee that the pooling equilibrium is always better, neither from an ex post nor from an ex ante point of view. In other words, which equilibrium gives the highest profit to the intermediary depends now on the particular distribution. However, in the cases in which the pooling equilibrium still dominates, our conclusions about over-lending and predatory lending remain true.

\(^9\)The separating equilibrium gives higher profits than the pooling equilibrium if the distribution is a truncated gamma with a pdf equal to

\[
f(\theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \theta^{\alpha-1} e^{-\frac{\theta}{\beta}} \frac{1}{\int_0^1 \left( \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} \right) dx}.
\]

with \( \alpha \) very small and \( \beta \) large.
6 Discussion and Applications

We analyzed a simple principal-agent model where, based on his expertise/experience, the principal has more information than the agent about some variables that affect the probability of reaching the high state of nature. In Section 4, we provided conditions under which the principal will not condition the transfer (or repayment) by the agent on the true information he owns. That is, conditionally on accepting to contract with the agent (a decision that depends on the true value of \( \theta \)), the principal will offer uniform (pooling) contracts. We showed that a necessary assumption for this result to hold is that there exists some level of complementarity between the information of the principal and the effort of the agent. Moreover, it is exactly this pooling that might make the principal be too aggressive by contracting with agents who, had they known the true \( \theta \), would have made lower effort. This result stems from the fact that, by committing to a pooling contract, the principal can impact the assessment of the agent regarding the marginal productivity of effort. That is, even though the agent is not fooled in expectations, all types take the same effort decision despite their different true marginal productivity of effort. Hence, a subset of types may end up with negative expected utility from contracting when conditioning on the principal’s information. When there is no asymmetric information about \( \theta \) the agent is able to predict the true marginal productivity of his own effort and too aggressive lending disappears. Moreover, Proposition 1 implies that lending is too conservative in this case. More information is not, however, Pareto improving. Indeed, there is always a subset of types below \( \theta^F \) that do not receive funding under complete information and hence get 0 utility, while they do get funding under incomplete information and obtain a positive utility. In contrast, types above \( \theta^F \) get higher utility under complete information because they are able to optimally choose their effort level.

Following the illustrative example used in the derivation of our results, lending is certainly a natural application of the framework in Section 4. However, since the ability of the agent to pay back his loan depends both on the information of the principal and on the effort that is exerted by the agent, financing of entrepreneurs by banks or venture capitalists appears to be more relevant than consumer credit. As has been advocated by Bond, Musto, and Yilmaz (2005) and Inderst (2008), for aggressive lending to emerge at equilibrium, one needs to model some costs on the side of the agent. Inderst (2008) does so by assuming that households face positive bankruptcy costs in case of default. Bond, Musto, and Yilmaz (2005) assume that there is a social cost of foreclosure. Our result does not hinge on the presence of any exogenous costs (as bankruptcy) but, in contrast, depends on the structural relationship between the action of the agent and the information of the principal.
The underlying cost stems from the *sub-optimal* level of effort chosen by the agent following the contractual offer of the principal.

Our framework and this structure of contracts is also reminiscent of the one at stake in creative or entertainment industries. Indeed, the outcome of talented artists will only become a success with the aid of inputs beyond their own inspiration. Among others, one can think of financing but also the experience of experts that have superior information regarding the tastes of potential consumers. The joint venture between an artist and his dealer is a simple illustration. The latter makes some investment, promotes the artist’s work within his social network and the two parties divide the gross revenue.\(^{10}\)

In Section 5 we have also shown that under certain conditions on the distribution of the principal’s information the previous results extend to more general contracting environments in which the investor can offer a positive payment even in the event that the project fails. Even though the signaling cost of the separating equilibrium is lower with more general contracts, it is still high compared to a simple pooling contract. Again, applying our analysis to the creative or entertainment industries, the principal can, in certain circumstances, provide some cash in advance to the artist which, generally, is proportional to her expectation of future revenues (success).

### A Appendix

**Proof of Lemma 1.** Suppose there is a separating equilibrium in which the investor makes an offer \(X(\theta)\) and finances all types above \(\theta^S\). Observing this offer, the entrepreneur interprets that the true parameter is \(\theta = X^{-1}(X(\theta))\) \((\beta(\theta, X, 1)) = 0\) if \(\theta < X^{-1}(X)\) and \(\beta(\theta, X, 1) = 1\) if \(\theta \geq X^{-1}(X)\) and chooses the effort level to maximize \(\theta e(R - X(\theta)) - \frac{\theta^2}{2}e^2\). Thus,

\[
e(\theta) = \theta \frac{R - X(\theta)}{e}.
\]

For this to be an equilibrium, it must be that a type \(\theta\) financier has no incentives to deviate:

\[
p(\theta, e(\theta))X(\theta) \geq p(\theta, e(\tilde{\theta}))X(\tilde{\theta}),
\]

for any \((\theta, \tilde{\theta}) \in [\theta^S, 1]^2\). When \(p(\theta, e) = \theta e\), this incentive constraint becomes

\[
e(\theta)X(\theta) = e(\tilde{\theta})X(\tilde{\theta}),
\]

\(^{10}\)Caves (2003) documents that the claim of the dealer can go up to 50%. Similarly, an author and his publisher, who support some fixed cost of promoting the novel, are linked by a revenue-sharing contract that can award the latter up to 58% of the gross profits that are generated. The way parties split revenue can depend on the identity of the agent once he is famous (asymmetric information is low) but the contracts are generally not idiosyncratic (pooling) for new artists.
or
\[ e(\theta)X(\theta) = K, \]
for \( K \) independent of \( \theta \). Since
\[ e(\theta) = \frac{\theta(R - X(\theta))}{c}, \]
we have that
\[ Kc = \theta(R - X(\theta))X(\theta). \tag{28} \]

Since the investor finances only types above \( \hat{\theta}^S \), it must be that her profits on that type are null:
\[ \pi(\hat{\theta}^S) = \hat{\theta}^S K - I = 0, \]
which implies that
\[ K = \frac{I}{\hat{\theta}^S}. \]
Replacing in (28) and solving for \( X(\theta) \) and \( e(\theta) \) respectively, we get (9) and (10). For these functions to exist it must be that for any \( \theta \),
\[ \frac{\theta \theta^S}{2} \geq \frac{4Ic}{R^2} \tag{29} \]
which is satisfied for any \( \theta \) if and only if it is satisfied for \( \hat{\theta}^S \). This entails that
\[ \hat{\theta}^S \geq \frac{\sqrt{4Ic}}{R} = \theta^F. \]

It can be shown that there are out-of-equilibrium beliefs such that any \( \hat{\theta}^S \geq \theta^F \) can be supported as an equilibrium.\footnote{For instance, put probability 1 on type 0 if a deviation is observed, which implies that effort is 0, so the deviation is not profitable.} Of course, the best separating equilibrium involves the lowest possible \( \hat{\theta}^S \), namely \( \theta^F \). \hfill \blacksquare

**Proof of Lemma 2.** Suppose there is a semi-separating equilibrium such that the investor finances types above some \( \hat{\theta}^{SP} \) and offers a separating contract \( X(\theta) \) for types on \( \hat{\Theta} \subset [\hat{\theta}^{SP}, 1] \) and a pooling contract \( \bar{X} \) for types on \( \hat{\bar{\Theta}} \). For this offer to be an equilibrium, it must be that it satisfies the investor’s incentive constraints:
\[ e(X(\theta))X(\theta) = e(\bar{X})\bar{X} = K \]
for all \( \theta \in \hat{\Theta} \), with
\[
\begin{align*}
e(X(\theta)) &= \frac{\theta(R - X(\theta))}{c} & \text{if } \theta \in \hat{\Theta} \\
e(\bar{X}) &= \frac{E(\theta|\theta \in \hat{\Theta})(R - \bar{X})}{c} & \text{if } \theta \in \hat{\bar{\Theta}}.
\end{align*}
\]
Since the investor finances only types above \( \hat{\theta}^{SP} \), it must be that her profits on that type are null:
\[ \pi(\hat{\theta}^{SP}) = \hat{\theta}^{SP} K - I = 0, \]

25
which implies that

\[ K = \frac{I}{\theta^{SP}}. \]

This implies that the contract must be

\[ X(\theta) = \frac{R}{2} \left( 1 \pm \sqrt{1 - \frac{4cI}{\theta^{SP} R^2}} \right), \]

\[ X = \frac{R}{2} \left( 1 \pm \sqrt{1 - \frac{4cI}{E(\theta|\theta \in \Theta)\theta^{SP} R^2}} \right) \]

Moreover, for this functions to exist, it must be that

\[ \frac{1}{2} \theta^{SP} \min\{\inf \hat{\Theta} \cup \{\theta|\theta \in \Theta\}\} \geq \frac{4c}{R^2}. \]

Again, the best semi-separating equilibrium involves the lowest possible \( \theta^{SP} \), namely \( \theta^{SP} \).

**Proof of Lemma 3.** Suppose there is a pooling equilibrium in which the investor offers \( X \) and finances types above \( \theta^{P} \). Then, when observing this offer the entrepreneur interprets that his type is \( \mathbb{E}(\theta|\theta > \theta^{P}) \)

\( (\beta(\theta|X, 1) = 0 \) if \( \theta < \theta^{P} \) and \( \beta(\theta|X, 1) = \frac{F(\theta) - F(\theta^{P})}{1 - F(\theta^{P})} \) if \( \theta > \theta^{P} \) \) and chooses his effort level to maximize his expected utility. From the moral hazard constraint we get \( e(X) \) defined by

\[ \mathbb{E}(\theta|\theta \geq \theta^{P})(R - X) = ce(X). \]

As before, we define

\[ K = e(X)X, \]

which is independent of \( \theta \). Thus,

\[ K = \mathbb{E}(\theta|\theta \geq \theta^{P}) \frac{R - X}{c} X. \]

Moreover, the profits on type \( \theta^{P} \) must be null, implying that

\[ K = \frac{I}{\theta^{P}} = \mathbb{E}(\theta|\theta \geq \theta^{P}) \frac{R - X}{c} X. \]

Solving for \( X \) this gives (11) and (12). Again, the best pooling equilibrium involves the lowest possible \( \theta^{P} \), namely \( \theta^{P} \).

**Proof of Proposition 7.** We proceed in two steps. We first show that the complete information contract is a PBE. Then, we show that it is unique. *(i)* **Existence.** Redoing the model with this specification, we get that the complete information contract is given by

\[ e_s^F(\theta) = \begin{cases} \frac{R}{2c} - \frac{\theta}{2} & \text{if } \theta \leq \frac{R}{2c}, \\ 0 & \text{if } \theta > \frac{R}{2c}, \end{cases} \]

and

\[ X_s^F(\theta) = \begin{cases} \frac{R}{2} + c\theta & \text{if } \theta \leq \frac{R}{2c}, \\ R & \text{if } \theta > \frac{R}{2c}, \end{cases} \]

26
and the investor finances all types above

\[
\theta^*_s = \begin{cases} 
2\sqrt{\frac{R}{2c}} - \frac{R}{2c} & \text{if } cI \leq \frac{R^2}{4} \\
\frac{2I}{4} & \text{if } cI > \frac{R^2}{4}.
\end{cases}
\]

We first show that it satisfies the investor’s incentive constraints (7) given the equilibrium beliefs. Note that when the entrepreneur observes \( X = R \) he must infer that his type is \( E(\theta|\theta > R/2c) \). Take two arbitrary types \( \theta \) and \( \tilde{\theta} \). We have to consider 2 cases. a) \( R/2c \geq \theta > \tilde{\theta} \).

The two incentive constraints write

\[
\frac{R + 2c\theta}{8c} \frac{R + 2c\tilde{\theta}}{2} \geq \left( \theta + \frac{R - 2c\tilde{\theta}}{4c} \right) \frac{R + 2c\tilde{\theta}}{4},
\]

\[
\frac{R + 2c\tilde{\theta}}{8c} \frac{R + 2c\theta}{2} \geq \left( \tilde{\theta} + \frac{R - 2c\theta}{4c} \right) \frac{R + 2c\theta}{4}.
\]

These two constraints can be rewritten as

\[
2(\theta - \tilde{\theta}) \frac{c}{2} \geq 0,
\]

\[
2(\tilde{\theta} - \theta) \frac{c}{2} \geq 0,
\]

which are both satisfied.

b) \( \theta > R/2c \geq \tilde{\theta} \).

Now, the two incentive constraints write

\[
\frac{R\theta}{2} \geq \left( \theta + \frac{R - 2c\tilde{\theta}}{4c} \right) \frac{R + 2c\tilde{\theta}}{4},
\]

\[
\frac{R + 2c\tilde{\theta}}{8c} \frac{R + 2c\theta}{2} \geq \frac{R\tilde{\theta}}{2}.
\]

These two inequalities can be rewritten as

\[
(R - 2c\tilde{\theta})(2c(\theta - \tilde{\theta}) - (R - 2c\theta)) \geq 0,
\]

\[
(R - 2c\tilde{\theta})^2 \geq 0,
\]

which are both satisfied because \( 2c\theta > R \geq 2c\tilde{\theta} \).

Since the financier finances only types above \( \theta^*_s \), all reimbursements \( \tilde{X} \in [0, X^F_s(\theta^*_s)] \) are out of equilibrium. Consider any out-of-equilibrium beliefs. Since they do not affect the effort decision, whatever the out-of-equilibrium beliefs, there is no profitable deviation.

(ii): Uniqueness. We again proceed in two steps. We first show that a pooling equilibrium only exists when \( cI \geq \frac{R^2}{4} \) and that it is such that \( X = R, \varepsilon = 0 \) and the investor finances only types above \( \frac{2I}{R} \). Next, we show that, if there exists a separating equilibrium for some subset of \( \tilde{\theta} \), it has the same structure as in the complete information equilibrium.

(a) Suppose first that there is a pooling equilibrium in which the investor finances types above some \( \theta^*_s \) by offering \( X \leq R \). The effort made by the entrepreneur given this offer is \( \varepsilon(X) = \frac{R - X}{2c} \) independent of his
beliefs about $\theta$. The profit of the investor in this equilibrium candidate for type $\theta$ is
\[
\max \left\{ 0; \left( \theta + \frac{R - X}{2c} \right) \frac{X}{2} - I \right\}.
\]

Consider a deviation to some $X' \neq X$. The investor’s profit under this deviation is
\[
\left( \theta + \frac{R - X'}{2c} \right) \frac{X'}{2} - I.
\]

Take a $\theta > \theta_s^P$. For this type, the deviation is not profitable only if
\[
X \in \arg \max \left\{ \left( \theta + \frac{R - X'}{2c} \right) \frac{X'}{2} \right\}.
\]

Note that the solution of (30) is independent of $\theta$ only if $2c\theta \geq R$ for all $\theta \geq \theta_s^P$ in which case we obtain the corner solution $X = R$. Therefore, suppose first that $2c\theta_s^P < R$, then there necessarily exists a profitable deviation for at least a $\theta > \theta_s^P$ and the pooling strategy cannot be an equilibrium. In contrast, assume that $2c\theta_s^P \geq R$, then $X = R$ is an optimal deviation. Therefore, the only pooling equilibrium has $X = R$, $e = 0$ and $2c\theta_s^P \geq R$.

Then the investor’s profit on type $\theta$ is
\[
\max \left\{ 0; \frac{\theta}{2} - I \right\},
\]
which implies that $\theta_s^P = \frac{2I}{c}$ (≡ $\theta_c^P$) and the condition $2c\theta_s^P \geq R$ becomes $cI \geq \frac{R^2}{4}$. This corresponds exactly to the pooling part of the semi-separating equilibrium under complete information described above. (b) Take any candidate for a separating equilibrium. It must be that it satisfies the investor’s incentive constraints given the optimal effort level. That is
\[
(\theta + e(\theta))X(\theta) \geq (\theta + e(\theta'))X(\theta'),
\]
with
\[
e(\theta) = \frac{R - X(\theta)}{2c}.
\]
This implies that
\[
X(\theta) \in \arg \max \left\{ \left( \theta + \frac{R - \bar{X}}{2c} \right) \bar{X} \right\}.
\]
But this just means that $X(\theta)$ must be the complete information equilibrium.

\[ \blacksquare \]

B Only the best equilibrium survives Grossman-Perry

We want to show that only the best pooling equilibrium survives the Grossman-Perry’s (Perfect Sequential Equilibrium) refinement. We proceed in two steps. We first show that the best pooling equilibrium is a PSE and, then, we show that it is the unique one.

28
Lemma 4 The best pooling equilibrium is a Perfect Sequential Equilibrium.

Proof. The best pooling equilibrium is characterized by

\[ X(\theta) = X^p = \frac{R}{2}, \quad e^p(\theta) = \mathbb{E}(\theta|\theta \geq \theta^p)\frac{R}{2c}. \]

The profit of the investor is

\[ \pi(\theta) = \max\{\theta\mathbb{E}(\theta|\theta \geq \theta^p)\frac{R^2}{4c} - I, 0\}, \]

and he finances all types above \( \theta^p \) defined such that

\[ \theta^p\mathbb{E}(\theta|\theta \geq \theta^p) = \frac{4Ic}{R^2}. \] (31)

We want to check that there is no consistent deviation. Consider a deviation to some \( X' \in [0, R], X' \neq R/2 \). Suppose that the entrepreneur’s beliefs after this deviation are that \( \theta \in T \subseteq [0, 1] \). Consistent beliefs imply that \( \mathbb{E}(\theta|X') = \mathbb{E}(\theta|\theta \in T) \) where the last expectation is computed using the prior distribution. The deviation is consistent if given these beliefs, it happens that the deviation is profitable for all types in \( T \) and not profitable for all types in the complement of \( T \). That is,

\[ \pi'(\theta) > \pi(\theta) \text{ if } \theta \in T, \]
\[ \pi'(\theta) \leq \pi(\theta) \text{ if } \theta \notin T, \]

where \( \pi'(\theta) = \max\{\theta e'(X')X' - I, 0\} \) and \( e'(X') = \mathbb{E}(\theta|X')(R - X')/c \). It is important to note that

\[ \frac{(R - X')X'}{c} < \frac{R^2}{4c} \] \hspace{1cm} (32)

for any \( X' \neq R/2 \).

Consider first a type such that \( \pi(\tilde{\theta}) > 0 \) and \( \pi'(\tilde{\theta}) > 0 \). Then, the deviation is profitable for this type if and only if

\[ \tilde{\theta} \left( \mathbb{E}(\theta|\theta \in T)\frac{R - X'}{c} X' - \mathbb{E}(\theta|\theta > \theta^p)\frac{R^2}{4c} \right) > 0. \]

Given (32), a necessary condition for this to happen is that \( T \) is such that

\[ \mathbb{E}(\theta|\theta \in T) > \mathbb{E}(\theta|\theta \geq \theta^p). \]

Note also that if the deviation is profitable for some type \( \tilde{\theta} > \theta^p \), it is also profitable for any type above \( \theta^p \). This implies that \( \tilde{T} \supseteq [\theta^p, 1] \), which contradicts the fact that

\[ \mathbb{E}(\theta|\theta \in T) > \mathbb{E}(\theta|\theta \geq \theta^p). \]

Thus, there is no consistent deviation and the equilibrium is a Perfect Sequential Equilibrium. \( \Box \)

Lemma 5 Only the best pooling equilibrium is a Perfect Sequential Equilibrium.
Proof. a) We start by showing that no other pooling equilibrium survives the refinement. Consider some pooling equilibrium with \( X(\theta) = \tilde{X} \neq R/2 \). Since this is a PBE we know that \( e(\tilde{X}) = E(\theta | \theta \geq \tilde{\theta})(R - \tilde{X})/c \), where \( \tilde{\theta} \) is the lowest type who gets financed and is defined such that

\[
\tilde{\theta} E(\theta | \theta \geq \tilde{\theta}) = \frac{Ic}{(R - \tilde{X})X} \geq \frac{4Ic}{R^2}.
\]

The profit of the investor in this equilibrium is

\[
\hat{\pi}(\theta) = \max\{\theta E(\theta | \theta \geq \tilde{\theta})(R - \tilde{X})/c, \tilde{X} - I, 0\}.
\]

Consider a deviation to \( X' = R/2 \) and suppose that following this deviation the entrepreneur’s (consistent) beliefs are that \( \theta \in T = (\theta^P, 1] \), with \( \theta^P \) defined as in (31). Note that \( \theta^P < \tilde{\theta} \). His effort is then

\[
e' = E(\theta | \theta \geq \theta^P)\frac{R}{2c},
\]

and the investor’s profit is

\[
\pi'(\theta) = \max\{\theta E(\theta | \theta \geq \theta^P)\frac{R^2}{4c} - I, 0\}.
\]

Hence, the deviation is consistent if

\[
\pi'(\theta) > \hat{\pi}(\theta) \quad \text{if} \ \theta \in [\theta^P, 1],
\]

\[
\pi'(\theta) \leq \hat{\pi}(\theta) \quad \text{if} \ \theta \notin [\theta^P, 1].
\]

Consider a type above \( \tilde{\theta} \), so that \( \hat{\pi}(\theta) > 0 \) and \( \pi'(\theta) > 0 \). For this type

\[
\pi'(\theta) - \hat{\pi}(\theta) = \theta \left( E(\theta | \theta > \theta^P)\frac{R^2}{4c} - E(\theta | \theta \geq \tilde{\theta})(R - \tilde{X})/c \right)
\]

\[
= \frac{\theta}{\tilde{\theta}} \left( \theta^P E(\theta | \theta > \theta^P)\frac{R^2}{4c} \tilde{\theta} - \tilde{\theta} E(\theta | \theta \geq \tilde{\theta})(R - \tilde{X})/c \right)
\]

\[
= \frac{\theta}{\tilde{\theta}} \left( \tilde{\theta} \frac{\theta^P I - I}{\theta^P} \right) > 0,
\]

where the last equality comes from the definitions of \( \theta^P \) and \( \tilde{\theta} \) and the inequality comes from the fact that \( \tilde{\theta} > \theta^P \). Thus, the deviation is profitable for any type in \([\tilde{\theta}, 1]\). Moreover, for types in \((\theta^P, \tilde{\theta}] \) we have that \( \pi'(\theta) > 0 = \hat{\pi}(\theta) \), so the deviation is profitable for all types above \( \theta^P \). Finally, for types in \([0, \theta^P] \) we have that \( \pi'(\theta) = \hat{\pi}(\theta) = 0 \) and the deviation is not profitable for these types.

Therefore, this deviation is profitable for any type in \( T \) and not profitable for any type not in \( T \) and is thus consistent. So, the equilibrium is not a Perfect Sequential Equilibrium. b) In order to show that no separating or semi-separating equilibrium survives the refinement, take the same deviation as before and the same argument goes through. An amendment needs to be done for the case of the best separating and semi-separating equilibria, because in those cases \( R/2 \) is an equilibrium offer. Then, take a deviation to \( \frac{R}{2}(1 + \epsilon) \) or \( \frac{R}{2}(1 - \epsilon) \) with \( \epsilon \to 0 \).

\[ \square \]

30
References


