Tax Increment Financing: Interaction Between Two Overlapping Jurisdictions.

by

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Abstract

This paper analyzes the interaction between two overlapping jurisdictions (a school district and a city), when one of them (the city) uses TIF (tax increment financing) to finance an improvement in the provision of its public good. Both jurisdictions levy taxes on property, and they provide different public goods. The city is not allowed to raise its tax rate to increase public good provision, but since the increment in the public good causes property values to increase, the city’s revenue is higher with the new level of public good. However, if this new revenue is not enough to finance the new public good level, then TIF allows the city to “capture”, for a limited period of time, all extra revenues the school district obtains due to the increase in property values. Then the city and the school district choose the public good level to satisfy their budget constraints with the goal of maximizing total property values. Two different results regarding efficiency of public good provision are obtained depending on the equilibrium concept used. With Nash behavior, the result is underprovision of the public good provided by the school district. The second, and more interesting, case is when the school district takes into account that its decisions affect the provision of the city’s public good. In this “leader-follower” equilibrium, where the school district plays the role of the leader, the results show that the provision of the school district public good depends on how the city’s public good is provided. If the city’s public good is underprovided (overprovided) then the school district’s public good is overprovided (underprovided).
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1 Introduction

In almost all cities there is a sharp contrast between nice neighborhoods and urban blight. This disparity among zones within the city leads to a problem that local governments face when a public improvement (e.g. repave the streets, broaden the sidewalks, etc.) in a deteriorated area has to be made. When a public improvement in a neighborhood is made, households find the place more attractive to live in, and then the property values in the area increase. This increase in the property tax base generates a higher revenue for the city. But, in general, this revenue is usually not enough to cover the cost of the public improvement. One of the ways that city officials can collect the extra revenue needed to cover the cost of the improvement is to increase the city’s property tax rate. The problem is that residents of the city outside the affected area do not benefit from the improvement, and face higher taxes. Then these people will oppose the improvement, and it may fail to be carried out.

To solve the problem cited above, a new method of financing public improvements in a city’s blighted zone, tax increment financing (TIF), has been widely used in the U.S. This method relies on a particular feature of the local public finance in the U.S.: taxation of the same property tax base by numerous overlapping jurisdictions. Separate fiscal entities, such as the city’s school district, also collect property taxes from the area, and the increase in the area’s property tax base following the improvement generates a tax windfall for the district. TIF allows the city to capture this tax windfall, and to use it to provide the extra revenue needed to make the public improvement. In this way, the city has TIF authority over the overlapping
jurisdictions in the improvement area (“TIF zone”), and collects all revenues generated by the increase in property values.

TIF’s widespread usage is not without controversy. Typical enabling statutes require the TIF zone to be a “blighted” area. But it seems that this is sometimes not accomplished, with relatively healthy areas used as a target for TIF. This apparent abuse sometimes leads jurisdictions whose revenue is captured by the implementing jurisdiction to complain that their funds are being “stolen”. Another question is whether TIF-financed improvements raise property values, as claimed by proponents.

All these issues are discussed by Chapman (1998) and Huddleston (1984). Some empirical literature has focused on the effects of TIF and on which forces lead to its adoption. Anderson (1990) shows that cities with high growth rates of property values are the ones that tend to adopt TIF. Other studies, like Man and Rosentraub (1998) and Dye and Merriman (1999) explore the effects of TIF on property value growth, but they arrive at opposite results. While the first study shows a positive effect of TIF on growth rates, the second one reaches the opposite conclusion.

Despite its broad use and the complexity of the TIF method, few theoretical papers analyze this important fiscal instrument. Dye and Sundberg (1998) and Donaghy, Elson and Knaap (1999) were the first two papers that try to analyze models of TIF. But none of them incorporates the important link between property values and the provision of public goods, which is the reason for TIF to exist.

Brueckner (2001) is the first paper that considers the connection between property values and public good provision in a model where TIF is used to finance a public improvement. In the study, it is shown that TIF is not always viable because it may not generate enough additional revenue. Viability is ensured only in the case where the public good is at least moderately underprovided. Another important conclusion of this paper is that the public good level chosen under TIF need not be efficient, with both over and underprovision being possible outcomes. In Brueckner’s model, jurisdictions whose tax revenue is captured by TIF play a passive role, and their public good level is held fixed in all the analysis.

The purpose of the present paper is to explore the interaction between jurisdictions under
TIF. In particular, the paper focuses on how public good provision in the jurisdiction whose tax revenue is captured (the school district) is affected by the presence of TIF. Since the question this paper wants to answer focuses on the relation between jurisdictions, spatial detail is suppressed by assuming that the entire city is affected by the public improvement. For political reasons, the city’s property rate is assumed to be fixed, so that a higher rate cannot be used to cover the cost of the public improvement. As a result, the city turns to TIF to generate the required tax revenue.\footnote{In Brueckner (2001), opposition to the city tax rate increase came from outside the improvement area, which receives no benefits from the improvement. With the improvement affecting the entire city, this type opposition does not emerge, so that the city’s tax rate is assumed to be fixed exogenously.}

In this framework, school district officials choose a public good level and a property tax rate to maximize total property value subject to their budget constraint,\footnote{See Brueckner (1979, 1982, 1983) for an explanation why this is a proper goal.} knowing that part of their revenue will be captured for a period of time. On the other hand, since its tax rate is fixed, the city chooses the largest possible level of public good that satisfies its budget constraint, a choice that again maximizes property value.

Two different kinds of equilibria are obtained. The first one is the result of assuming that the school district takes the city actions as given when choosing its property tax rates and public good level. The result of this exercise gives a Nash equilibrium, where the school district’s public good is underprovided regardless of whether the city’s public good is under or overprovided. The reason for this result is that the school district is not allowed to use part of their revenues (the part captured by the city) during a period of time, so that their revenue is not enough to cover the costs of the efficient level of the public good.

The second kind of equilibrium is more interesting. It is the result of realizing that the school district, by choosing a public good level and property tax rate, influences the city’s public good level decision. Then a kind of leader-follower equilibrium is obtained, where the school district plays the role of the leader. In this case, the efficiency of school district’s choice depends on the level of the public good chosen by the city. The school district underprovides (overprovides) its public good when the city’s public good level is overprovided (underprovided).

The next section of the paper establishes the set up of the model. Section three discusses the different kinds of equilibria that are obtained, and section four concludes.
2 The Model

2.1 Setup

As in Brueckner (2001), the model considers the connection between property values and public good provision. To do so, it is supposed that the city is composed of only one neighborhood and that it provides a single public good $z$. The total cost of providing the public good is $C(z)$, where the cost function $C(z)$ is convex. Residents of the city also consume another public good, $s$, provided by a coterminous and separate jurisdiction (the school district). The cost of providing $s$ is given by $K(s)$, also a convex function.

City residents are assumed to be identical, with preferences representable by utility function $U(.)$. They are also assumed to rent the houses they occupy. Let $q_i$ denote the vector of attributes of house $i$. Then the utility of house $i$’s occupant is given by $U(x_i, q_i, z, s)$, where $x_i$ is the individual’s consumption of a numeraire private good. The budget constraint for house $i$’s resident is given by $x_i + P_i = y$, where $y$ is the common income level of city residents and $P_i$ the rent payment for house $i$.

Following Wheaton (1977) and Brueckner (1979, 1982), the rent payment $P_i$ is determined using a “bid rent” approach. This approach assumes that residents of the city are freely mobile, and then they enjoy a fixed utility level equal to that available elsewhere in the economy. Then $U(y - P_i, q_i, z, s) = \bar{U}$, where $\bar{U}$ is the common utility level and where private good consumption is eliminated by using the budget constraint. This condition implicitly defines $P_i$ as a function of $z$, $q_i$, and $s$, with partial derivatives $\partial P_i / \partial z = U_t^z / U_t^x$, $\partial P_i / \partial q_i = U_t^q / U_t^x$, and $\partial P_i / \partial s = U_t^s / U_t^x$. Since marginal utilities are positive, the rent for house $i$ increases as the public good levels or the housing attributes increase.

Holding the housing attributes fixed, rent can be written as a function of both public goods,

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3 In Brueckner’s model there are two city neighborhoods and a coterminous jurisdiction that plays a passive role. In order to facilitate the analysis of the interaction between the city and the coterminous jurisdiction, the neighborhood where the improvement is not made is eliminated from the model.

4 For this to happen, it is needed that the city is small with respect to the rest of the economy.

5 Where the superscripts denote partial derivatives and the subscripts denote the particular household.

6 Notice that this assumption rules out real estate investment in response to a public improvement. As shown by Brueckner (2001), the results of this kind of analysis are unaffected if the $q$’s change in response to changes in the levels of $z$ and $s$. 

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Then total rent in the city is given by

\[ R(z, s) \equiv \sum_i P_i(z, s) \]  

(1)

and the partial derivatives of the total rent are given by

\[ \frac{\partial R(z, s)}{\partial z} = \sum_i MRS^{z,x}_i ; \quad \frac{\partial R(z, s)}{\partial s} = \sum_i MRS^{s,x}_i, \]

(2)

where \( MRS^{z,x}_i = U^z_i / U^{x}_i \) and \( MRS^{s,x}_i = U^s_i / U^{x}_i \).

The value of the houses in the city depends on the rents they generate and on the property taxes they pay. In the model, it is assumed that both jurisdictions, the city and the school district, finance their provision of public goods with property tax revenue. The tax rate is denoted \( \tau \) for the city, and \( t_s \) for the school district.

Using (1) and (2), the social optimality conditions are obtained for use as a benchmark in the analysis that follows. Optimality obtains when both the school district and the city set their public good levels and tax rates so as to maximize the total value of property.\(^7\) Notice that since value is given by capitalized rent minus total taxes, which equal public good costs, total property value can be written as \( (R(z, s) - C(z) - K(s))/r \). The first order conditions for the choice of \( z \) and \( s \) are then

\[ \frac{\partial R(z, s)}{\partial z} = C'(z) \]

(3)

\[ \frac{\partial R(z, s)}{\partial s} = K'(s). \]

(4)

Thus, a marginal change in rent from an increase in the public good should equal the cost of the extra public good. Recalling (2), conditions (3) and (4) reduce to the Samuelson optimality

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\^7\]Brueckner (1979, 1982, 1983) shows that since the utilities of the city residents are fixed the proper goal for the government is to choose the public good level in the interest of property owners. Then this level has to maximize the total value of property in the city.
conditions $\sum_i MRS_i^{z,x} = C'(z)$ and $\sum_i MRS_i^{s,x} = K'(s)$, which say that the marginal social benefit from an increase in the public good equals marginal cost.

For future reference, the public good levels that satisfy equations (3) and (4) (the socially optimal levels) are denoted by $z^*$ and $s^*$. The public good is said to be underprovided (overprovided) when its level is smaller (bigger) than the socially optimal level. Since $\partial^2 R(z, s)/\partial z^2 < 0$ and $C''(z) \geq 0$ hold, underprovision (overprovision) of $z$ implies $\partial R(z, s)/\partial z > (\langle,)C'(z)$. Similarly, since $\partial^2 R(z, s)/\partial s^2 < 0$ and $K''(s) \geq 0$ hold, underprovision (overprovision) of $s$ implies $\partial R(z, s)/\partial s > (\langle,)K'(s)$.

The goal of the paper is to analyze the school district’s behavior when the city is using TIF to finance a public improvement, defined as a marginal increase in $z$. Under TIF, the city is not able to increase the property tax rate. It uses the increment in its own tax revenue, due to the increase in property values, to finance the improvement of the public good. Since the increase in revenue is usually not sufficient to cover the cost of the improvement, the city is allowed to capture all extra revenue accruing to other overlapping jurisdictions (in this case the school district) because of the increment in property values. The provision of the city’s public good is then $z_1 = z_0 + \Delta$, where $z_0$ is the initial level of public good and $\Delta$ is the improvement, and the property tax rate is fixed at $\tau = \tau_0$.

The problem is modeled in continuous time. The city makes the improvement (increasing the provision of the public good) at time $T_0 \geq 0$, and is able to collect extra revenues from the school district until time $T_1 > T_0$. After $T_1$, TIF authority ceases, and the city is no longer able to capture any school district revenue. School district decisions are made at time zero.

Since this is a model with perfect foresight, the change in $z$ at time $T_0$ is considered when the property values at time zero are calculated. As shown in the appendix, the total value of houses is then given by

$$0 \leq t \leq T_0, \quad V(t) = \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0 + t_s + r)(T_0 - t)}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-(\tau_0 + t_s + r)(T_0 - t)} \quad (5)$$

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8 This can be justified by political reasons, such as that the citizens can vote against the improvement if the tax rate is increased.

9 The calculation follows Anderson (1986). See the Appendix for the calculations.
\[ t \geq T_0, \quad V(t) = \frac{R(z_1, s)}{\tau_0 + t_s + r}. \quad (6) \]

where \( r \) is the interest rate.

Because the city’s public good level is constant at \( z_1 \) after \( T_0 \), the property value solution in (6) comes from solving the equation \[ V = \left[ R(z_1, s) - (\tau_0 + t_s)V \right]/r. \] On the other hand, since \( z \) changes at \( T_0 \), the solution for \( t < T_0 \) is more involved. However, (5) shows that property value in this case is a weighted average of \[ R(z_0, s)/(\tau_0 + t_s + r) \] and \[ R(z_1, s)/(\tau_0 + t_s + r), \] with the weights depending on \( t \).

2.2 Budget Constraints

Assuming perfect capital markets, which allows the city to borrow against future tax revenue, the city’s budget constraint requires that the present value of revenue equals the present value of its cost. The constraint is

\[
\int_{0}^{T_0} \tau_0 \left[ \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0 + t_s + r)(T_0 - t)}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-r(t_0 + t_s + r)(T_0 - t)} \right] e^{-rt} dt
\]

\[
+ \int_{T_0}^{T_1} \frac{R(z_1, s)}{\tau_0 + t_s + r} + t_s \left( \frac{R(z_1, s)}{\tau_0 + t_s + r} - \frac{R(z_0, s)}{\tau_0 + t_s + r} \right) e^{-rt} dt + \int_{T_1}^{\infty} \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-rt} dt
\]

\[ = \int_{0}^{T_0} C(z_0) e^{-rt} dt + \int_{T_0}^{\infty} C(z_1) e^{-rt} dt. \quad (7) \]

The budget constraint shows that between 0 and \( T_0 \) the city collects its own revenue (i.e., \( \tau_0 \) times the value of houses at that moment of time). Between \( T_0 \) and \( T_1 \) the city not only collects its own revenue, but also collects the extra revenue the school district is getting due to the increase in the house value.\(^{10}\) Then from \( T_1 \) onward, the city returns to collecting just its

\(^{10}\) Notice that in the absence of the increment in the city’s public good, the total value of houses is given by \[ V(t) = \int_{0}^{\infty} [R(z_0, s) - (\tau_0 + t_s)V(t)] e^{-rt} d\tau = R(z_0, s)/(\tau_0 + t_s + r). \]
own revenue. This total revenue has to cover the present value of the cost of provision of the public good (the right hand side of (7)).

Solving and rearranging (7), the following form for the city’s budget constraint is obtained:

$$\tau_0 \left[ \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-rT_0}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-rT_0} \right]$$

$$+ \left[ \frac{R(z_1, s)}{\tau_0 + t_s + r} - \frac{R(z_0, s)}{\tau_0 + t_s + r} \right] \left[ \frac{r_T_0}{(\tau_0 + t_s)} \left( e^{(r_0 + t_s)T_0} - 1 \right) e^{-(r_0 + t_s)T_0} + t_s (e^{-rT_0} - e^{-rT_1}) \right]$$

$$= C(z_0)(1 - e^{-rT_0}) + C(z_1)e^{-rT_0}.$$ \hspace{1cm} (8)

On the other hand, the school district budget constraint is given by

$$\int_0^{T_0} t_s \left[ \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0 + t_s + r)(T_0-t)}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-(\tau_0 + t_s + r)(T_0-t)} \right] e^{-rt} dt$$

$$+ \int_{T_0}^{T_1} t_s \frac{R(z_0, s)}{\tau_0 + t_s + r} e^{-rt} dt + \int_{T_1}^{\infty} t_s \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-rt} dt = \int_0^{\infty} K(s)e^{-rt} dt.$$ \hspace{1cm} (9)

Thus, during the period of time TIF is operating (between $T_0$ and $T_1$), the school district revenues are calculated taking into account the “without TIF” property value (the extra revenue is captured by the city). Once TIF ends, the school district is allowed to get the extra revenues from the property value increase. Again, this revenue has to cover the cost of provision of the public good (the right hand side of (9)), in present value terms.

Solving and rearranging (9) gives

$$t_s \left\{ \frac{[R(z_0, s)(1 - e^{-rT_1}) + R(z_1, s)e^{-rT_1}]}{\tau_0 + t_s + r} + \frac{(R(z_1, s) - R(z_0, s))}{(\tau_0 + t_s + r)} e^{-(\tau_0 + t_s + r)T_0} \right\} e^{r_0 + t_s - 1} = K(s).$$ \hspace{1cm} (10)
2.3 Optimization Problems

The optimization problem for the school district is to choose a property tax rate and a public good level to maximize total property value subject to its budget constraint. In this choice, it either views $z_1$ as parametric, or it takes into account the effect of its choices on $z_1$. Once the general form of the first order conditions is derived, both types of behavior are analyzed.

Formally, the school district problem is to maximize property value at time zero,

$$\max_{(s,t_0)} \left\{ \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0 + t_0 + r)T_0}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-(\tau_0 + t_0 + r)T_0} \right\}$$

subject to the budget constraint (10).

To facilitate the analysis, the case where $T_0$ approaches to zero is considered. This case corresponds to a situation where the interval between the school district’s decision time and the start of TIF becomes negligible. The reason for setting up the model with $T_0 \geq 0$ (instead of imposing $T_0 = 0$ at the outset) is to highlight the fact that the city takes account of the fixed fiscal environment, which includes an existing level of $s$ set at time zero, in selecting $z_1$ (a decision that is made at time $T_0$). Allowing $T_0$ to approach zero, which shortens the interval between the time at which $s$ is set and the onset of TIF, simplifies the analysis.

With $T_0 = 0$, the Lagrangian for the school district’s optimization problem is

$$\mathcal{L} = \frac{R(z_1, s)}{\tau_0 + t_s + r} + \lambda \left\{ t_s \left( \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-rT_1}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-rT_1} \right) - K(s) \right\}. \quad (11)$$

The first order conditions are\(^\text{11}\)

$$\frac{\partial \mathcal{L}}{\partial s} = \frac{1}{\tau_0 + t_s + r} \left( \frac{\partial R(z_1, s)}{\partial s} + \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial s} \right)$$

$$+ \lambda \left\{ t_s \left[ \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-rT_1}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-rT_1} \right] - K'(s) \right\} = 0 \quad (12)$$

\(^{11}\)Since all functions in the problem are continuous, the same first order conditions would be obtained if $T_0$ were set to zero after rather than before differentiation. In other words, $\lim_{T_0 \to 0} \partial \mathcal{L}(s, t_0, T_0)/\partial s = \partial \mathcal{L}(s, t_0, \lim_{T_0 \to 0} T_0)/\partial s = \partial \mathcal{L}(s, t_0, 0)/\partial s.$
\[
\frac{\partial \mathcal{L}}{\partial t_s} = \frac{R(z_1, s)}{(\tau_0 + t_s + r)^2} + \frac{1}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} + \lambda \left\{ \frac{(\tau_0 + r)}{(\tau_0 + t_s + r)^2} \left[ (1 - e^{-rT_1}) R(z_0, s) + e^{-rT_1} R(z_1, s) \right] + \frac{t_s e^{-rT_1}}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} \right\} = 0 \quad (13)
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{t_s}{\tau_0 + t_s + r} \left[ (1 - e^{-rT_1}) R(z_0, s) + e^{-rT_1} R(z_1, s) \right] - K(s) = 0. \quad (14)
\]

In order to further simplify the discussion, a separability assumption on preferences is imposed. With the utility function assumed to be additively separable, \(\frac{\partial R(z, s)}{\partial s}\) is independent of \(z\), so that \(\frac{\partial R(z_0, s)}{\partial s}\) and \(\frac{\partial R(z_1, s)}{\partial s}\) are equal. Imposing this simplification in (12) and (13), the following optimality condition is obtained:

\[
\left[ (R(z_1, s)e^{-rT_1} + R(z_0, s)(1 - e^{-rT_1})) \right] \frac{(\tau_0 + r)}{(\tau_0 + t_s + r)^2} + \frac{t_s R(z_1, s)}{(\tau_0 + t_s + r)^2} - \frac{t_s (1 - e^{-rT_1})}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} \frac{\partial R(z_1, s)}{\partial s} = K'(s) \left[ \frac{R(z_1, s)}{(\tau_0 + t_s + r)^2} - \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} \right]
\]

\[
= K'(s) \left[ \frac{R(z_1, s)}{(\tau_0 + t_s + r)^2} - \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} \right]
\]

The interpretation of this condition is left for the next section.

On the other hand, the city chooses the largest public good level satisfying its budget constraint (8). In this decision, the city behaves in Nash fashion, viewing \(s\) as parametric. The city’s behavior can be better understood with the help of Figure 1. To generate the curve shown in Figure 1, the city’s budget constraint with TIF (equation(8)) can be used to derive the budget-balancing \(\tau\):\(^{12}\)

\(^{12}\)The calculations are made for the case where \(T_0\) equals zero.
\[ \tau = \left( t_s + r \right) C(z) - t_s (1 - e^{-rT_1}) \left( R(z, s) - R(z_0, s) \right) \frac{R(z, s) - C(z)}{R(z, s) - C(z)}, \quad z \geq z_0. \] (16)

From (16), the slope of the “TIF curve” relating \( \tau \) to \( z \) for \( z \geq z_0 \) is given by

\[ \frac{d\tau}{dz} = \left( \tau + t_s + r \right) \left( C'(z) - \frac{\left( \tau + t_s (1 - e^{-rT_1}) \right) \partial R(z, s)}{\tau + t_s + r} \right). \] (17)

The sign of (17) is given by the sign of

\[ \Gamma(z) = C'(z) - \frac{\left( \tau + t_s (1 - e^{-rT_1}) \right) \partial R(z, s)}{\tau + t_s + r}. \] (18)

It can be shown that this expression changes sign at most once as \( z \) increases.\(^{13}\) When the sign changes, it goes from negative to positive. Alternatively, the sign could be positive for all \( z \geq z_0 \). Figure 1, which shows the TIF curve as initially downward sloping and then upward sloping, corresponds to the first case.

As shown in Brueckner (2001), viability of TIF requires that the TIF curve is downward sloping at \( z = z_0 \), as shown. To see the reason, note that combinations of \( z \) and \( \tau \) that lie above the curve generate a budget surplus, while those below the curve generate a budget deficit. It follows that, when the TIF curve is downward sloping at \( z = z_0 \), an increase in \( z \) above \( z_0 \) with \( \tau \) held fixed at \( \tau_0 \) generates a budget surplus. TIF is then viable, generating enough revenue to pay for the public improvement. By contrast if the TIF curve were upward sloping at \( z = z_0 \), an increase in \( z \) would yield a budget deficit, indicating the TIF is not viable.

Since the city’s tax rate is held fixed, the goal of maximizing property value is achieved by selecting the largest possible \( z \) consistent with a balanced budget. This value, \( z_1 \), lies at the

\[ \frac{\partial \Gamma(z)}{\partial z} > 0 \] holds when \( \Gamma(z) \leq 0 \). Therefore, since \( \partial \Gamma(z)/\partial z > 0 \) holds when \( \Gamma(z) \leq 0 \), it follows that \( \Gamma(z) \) changes sign once, from negative to positive, provided that \( \Gamma(0) < 0 \). The latter condition requires that \( \partial R(0, s)/\partial z \) is substantially larger than \( C'(0) \), indicating that the marginal benefit of the first unit of \( z \) greater exceeds its cost. Otherwise, \( \Gamma(z) > 0 \) holds for all \( z \geq z_0 \).
point where the horizontal line through \((z_0, \tau_0)\) intersects the TIF curve for the second time, as shown in Figure 1. A key property of this solution is that the TIF curve is upward sloping at \(z_1\), with (18) greater than zero. Since the term multiplying \(\partial R(z, s)/\partial z\) in (18) is less than one, it is easy to see that this requirement is consistent with either \(\partial R(z, s)/\partial z \geq C'(z)\) or \(\partial R(z, s)/\partial z \leq C'(z)\). Thus, under TIF, the city’s public good can be under or overprovided.

3 Analysis of the Optimality Conditions

3.1 Nash Equilibrium

From the discussion above it follows that, since the city chooses the largest \(z\) satisfying its budget constraint, the only optimality condition to analyze is equation (15), which comes from the school district problem.

In this section it is assumed that the school district takes the city’s decisions as given. Then it does not take into account, when deciding the level of public good to provide, how its choices affect the city’s public good provision. In this case \(\partial z_1/\partial s\) and \(\partial z_1/\partial t_s\) are both zero, and the following condition is obtained:

\[
\left[ (R(z_1, s)e^{-rT_1} + R(z_0, s)(1 - e^{-rT_1})) (\tau_0 + r) + t_s R(z_1, s) \right] \frac{\partial R(z_1, s)}{\partial s} = (\tau_0 + t_s + r) R(z_1, s) K'(s). \tag{19}
\]

Notice that this condition is of the form \(\alpha \partial R(z_1, s)/\partial s = \beta K'(s)\), where \(\alpha\) is equal to the term in brackets on the left hand side and \(\beta\) is equal to \((\tau_0 + t_s + r) R(z_1, s)\). Then, to see if the school district public good is under or overprovided, the relation between \(\alpha\) and \(\beta\) has to be analyzed. In particular if \(\beta - \alpha > (0)\) the public good is underprovided (overprovided), and it is optimally provided when the difference is zero. Substituting from (19), it follows that

\[
\beta - \alpha = (\tau_0 + r)(1 - e^{-rT_1})(R(z_1, s) - R(z_0, s)) > 0, \tag{20}
\]

since \(R(z_1, s) > R(z_0, s)\). Therefore
so that the school district public good is underprovided. Summarizing yields,

**Proposition 1** When the city’s decisions are taken as fixed by the school district, the school district’s public good is underprovided.

To see why the school district underprovides its public good, consider again the optimization problem represented by (11). With \( z_1 \) viewed as fixed under Nash behavior, the school district chooses \( s \) to maximize total value subject to its budget constraint, represented by the second part of (11). If \( z_0 \) in that constraint were actually equal to \( z_1 \), then it is easily seen that the school district’s choice would be efficient, with \( s \) satisfying \( \frac{\partial R(z_1, s)}{\partial s} = K'(s) \). This fact simply repeats the conclusions already stated in (3) and (4). However, because \( z_0 \) is less than \( z_1 \), the tax revenue accruing to the school district for any given \( s \) is smaller than the amount that generates this efficient outcome. As a result, the school district’s chosen \( s \) is suboptimal, lying below the level that maximizes total property value.

### 3.2 School District as a Leader

The second, and more interesting, case is when the school district, choosing \( s \) and \( t_s \), considers the effects its decisions have on the city’s public improvement. Rather than being zero, the relevant values of \( \frac{\partial z_1}{\partial s} \) and \( \frac{\partial z_1}{\partial t_s} \) must be substituted into the school district’s first order conditions. These effects are obtained by differentiating the city’s budget constraint.

Then, differentiating the city’s budget constraint (equation (6)) with respect to \( s \) and \( t_s \), the following equations are obtained\(^{14}\)

\[
\frac{\tau_0}{\tau_0 + t_s + r} \left( \frac{\partial R(z_1, s)}{\partial s} + \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial s} \right) + \frac{t_s(1 - e^{-rT_1})}{\tau_0 + t_s + r} \left( \frac{\partial R(z_1, s)}{\partial s} + \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial s} - \frac{\partial R(z_0, s)}{\partial s} \right) - C'(z_1) e^{-rT_0} \frac{\partial z_1}{\partial s} = 0,
\]

\(^{14}\)Again, calculations are made for the case where \( T_0 \) equals zero.

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\[ -\frac{\tau_0}{(\tau_0 + t_s + r)^2} R(z_1, s) + \frac{\tau_0}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} \frac{t_s(1 - e^{-rT_1})}{(\tau_0 + t_s + r)^2} (R(z_1, s) - R(z_0, s)) \]

\[ + \frac{t_s(1 - e^{-rT_1})}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} \frac{\partial z_1}{\partial t_s} + \frac{(R(z_1, s) - R(z_0, s))}{\tau_0 + t_s + r}(1 - e^{-rT_1}) - C'(z_1) \frac{\partial z_1}{\partial t_s} = 0. \] (23)

Again, using the assumption of separability in preferences and arranging, the following equations for \( \frac{\partial z_1}{\partial s} \) and \( \frac{\partial z_1}{\partial t_s} \) are obtained from (22) and (23):

\[ \frac{\partial z_1}{\partial s} = \frac{1}{\Gamma(z_1)} \left[ \frac{\tau_0}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial s} \right], \] (24)

\[ \frac{\partial z_1}{\partial t_s} = \frac{1}{\Gamma(z_1)} \left[ \frac{(R(z_1, s) - R(z_0, s))}{(\tau_0 + t_s + r)^2}(1 - e^{-rT_1})(\tau_0 + r) - \frac{\tau_0}{(\tau_0 + t_s + r)^2} R(z_1, s) \right], \] (25)

where

\[ \Gamma(z_1) = C'(z_1) - \frac{(\tau_0 + t_s(1 - e^{-rT_1}))}{\tau_0 + t_s + r} \frac{\partial R(z_1, s)}{\partial z_1} > 0, \] (26)

and where the inequality follows from the discussion of Figure 1.

Given (26), it follows that \( \frac{\partial z_1}{\partial s} > 0 \). Thus, an increase in the school district public good increases the city’s improvement. The increase in \( s \) leads to higher property values, which leave the city with more revenue to capture. As a result the city’s public improvement is increased. On the other hand, the sign of \( \frac{\partial z_1}{\partial t_s} \) cannot be determined. While an increase in the school district property tax rate increases the amount of extra revenue the city collects, a positive effect, a higher tax rate also reduces property values, and this is a negative effect. Thus, the final effect on \( z_1 \) is ambiguous.

Substituting (24) and (25) in the first order condition (15), the following condition is obtained:
After some substitutions and arrangements, equation (27) collapses to

\[
\left\{ \left( R(z_1, s)e^{-rT_1} + R(z_0, s)\left(1-e^{-rT_1}\right) \right) \frac{(\tau_0 + r)}{\tau_0 + t_s + r} + \frac{t_s}{\tau_0 + t_s + r} R(z_1, s) \right. \\
- \frac{t_s(1-e^{-rT_1})}{\Gamma(z_1)} \left[ \frac{(R(z_1, s) - R(z_0, s))}{(\tau_0 + t_s + r)^2} \right] (1-e^{-rT_1})(\tau_0 + r) - \frac{\tau_0}{(\tau_0 + t_s + r)^2} R(z_1, s) \right\} \frac{\partial R(z_1, s)}{\partial z_1} \\
+ \frac{\tau_0}{\Gamma(z_1)} \left[ \frac{t_s e^{-rT_1} R(z_1, s)}{(\tau_0 + t_s + r)^2} + \frac{R(z_1, s)e^{-rT_1} + R(z_0, s)(1-e^{-rT_1})}{(\tau_0 + t_s + r)^2} \right] (\tau_0 + r) \frac{\partial R(z_1, s)}{\partial z_1} \right\} \frac{\partial R(z_1, s)}{\partial s} \\
= K'(s) \left\{ R(z_1, s) - \frac{1}{\Gamma(z_1)} \left[ (R(z_1, s) - R(z_0, s)) \frac{(1-e^{-rT_1})(\tau_0 + r)}{\tau_0 + t_s + r} - \frac{\tau_0 R(z_1, s)}{\tau_0 + t_s + r} \right] \frac{\partial R(z_1, s)}{\partial z_1} \right\} (27)
\]

After some substitutions and arrangements, equation (27) collapses to

\[
\left\{ \left( R(z_1, s)e^{-rT_1} + R(z_0, s)\left(1-e^{-rT_1}\right) \right) \frac{(\tau_0 + r)}{\tau_0 + t_s + r} \frac{R(z_1, s)}{\partial z_1} \right. \\
+ \frac{t_s R(z_1, s)}{\tau_0 + t_s + r} \left[ C'(z_1) - (1-e^{-rT_1}) \frac{\partial R(z_1, s)}{\partial z_1} \right] \right\} \frac{\partial R(z_1, s)}{\partial s} \\
= \left\{ \left( R(z_1, s)e^{-rT_1} + R(z_0, s)\left(1-e^{-rT_1}\right) \right) \frac{(\tau_0 + r)}{\tau_0 + t_s + r} \right. \\
+ \left( R(z_1, s)e^{-rT_1} + R(z_0, s)(1-e^{-rT_1}) \right) \frac{t_s e^{-rT_1}}{\tau_0 + t_s + r} \frac{R(z_1, s)}{\partial z_1} \right\} K'(s). \tag{28}
\]

As in the previous case, this condition is of the form \(\gamma \partial R(z_1, s)/\partial s = \theta K'(s)\), where \(\gamma\) and \(\theta\) are the terms in brackets on the left hand side and the right hand side respectively. To see whether the school district public good is under or overprovided, the difference between \(\theta\) and \(\gamma\) is calculated:

\[
\theta - \gamma = R(z_1, s) \left( C'(z_1) - \frac{\partial R(z_1, s)}{\partial z_1} \right) \left( 1 - \frac{t_s}{\tau_0 + t_s + r} \right)
\]
\[-(R(z_1, s)e^{-rT_1} + R(z_0, s)(1 - e^{-rT_1})) \left( \frac{(\tau_0 + r)}{\tau_0 + t_s + r} \left(C'(z_1) - \frac{\partial R(z_1, s)}{\partial z_1} \right) \right) \]

\[\left( C'(z_1) - \frac{\partial R(z_1, s)}{\partial z_1} \right) \left( \frac{(\tau_0 + r)}{\tau_0 + t_s + r}(1 - e^{-rT_1}) (R(z_1, s) - R(z_0, s)) \right). \]

(29)

Since the last term on the right hand side is positive \((R(z_1, s) > R(z_0, s))\), the sign of equation (29) depends on the sign of \((C'(z_1) - \partial R(z_1, s)/\partial z_1)\). That is, the provision of the school district public good depends on how the city’s public good is provided. The following relations hold

\[\frac{\partial R(z_1, s)}{\partial z_1} < C'(z_1) \Rightarrow \frac{\partial R(z_1, s)}{\partial s} > K'(s)\]

\[\frac{\partial R(z_1, s)}{\partial z_1} > C'(z_1) \Rightarrow \frac{\partial R(z_1, s)}{\partial s} < K'(s)\]

\[\frac{\partial R(z_1, s)}{\partial z_1} = C'(z_1) \Rightarrow \frac{\partial R(z_1, s)}{\partial s} = K'(s).\]

Since equation (26) holds, any of the three results could be possible. Summarizing yields,

**Proposition 2** When the school district takes into account the effects of its decisions on the city’s public improvement, three possible equilibria are present. They are: underprovision of the school district’s public good and overprovision of the city’s public good, overprovision of the school district’s public good and underprovision of the city’s public good, and efficient provision of both public goods. In other words, overprovision or underprovision of both public goods are not possible equilibria.

To understand the results in the Proposition, recall that the school district’s goal is to choose \(s\) so as maximize total property value. In addition, recall that, for a given \(s\), the city’s public good level can be either under or overprovided, being too low or too high relative to the level that maximizes total property value. Acting as a leader, the school district can adjust its own choice to offset the inefficient tendencies of the city, and this fact explains Proposition 2. In particular, if \(z_1\) ends up being too high, with \(\partial R(z_1, s)/\partial z_1 < C'(z_1)\) holding, then the school district will have compensated by setting \(s\) below the level that maximizes total value, with \(\partial R(z_1, s)/\partial s > K'(s)\). Conversely, if \(z_1\) ends up too low, with \(\partial R(z_1, s)/\partial z_1 > C'(z_1)\) holding,
then the school district will have compensated by setting $s$ above the value-maximizing level, with $\partial R(z_1, s)/\partial s < K'(s)$. Finally, if $z_1$ is chosen efficiently, then there is no need for the school district to compensate, so that $s$ is also chosen efficiently. In this way, the school district as a leader attempts to steer the joint choice of $z_1$ and $s$ toward an outcome that maximizes total property value.

Proposition 2 is, of course, incomplete because it does not specify the conditions under which the three different cases will arise. In principle, the likely parameter ranges leading to the different cases could be revealed by comparative-static analysis of the equilibrium conditions. For example, if comparative-static analysis showed $z_1$ to be decreasing in the interest rate $r$, then the first case above (overprovision of $z_1$, underprovision of $s$) would be associated with low $r$ values, while the second case would be associated with high $r$ values. Unfortunately, however, the complexity of the equilibrium conditions makes comparative-static analysis impractical. As a result, the only way to explore the connection between parameter values and the different equilibrium outcomes would be through numerical analysis.

4 Conclusion

Although use of tax increment financing has become widespread in the U.S. in recent years, little effort has been devoted to economic analysis of TIF. Building on Brueckner (2001), this paper analyzes an issue that is not considered in previous work: interaction between overlapping jurisdictions in the presence of TIF. Recognizing that jurisdictions whose tax revenue is expropriated through TIF may not be passive players, the paper analyzes the choice of property taxes and public spending by a school district that expects a portion of its tax revenue to be captured by the city via TIF. With TIF now widely used, this kind of sophisticated behavior on the part of school districts and other affected jurisdictions is likely, and the paper provides a needed analysis of it.

Two different results regarding efficiency of public good provision are obtained depending on the equilibrium concept used. First, the Nash equilibrium is considered, where the school district takes the city’s decisions as given. Then the result is underprovision of the public good provided by the school district. The second, and more interesting case, is when the school
district recognizes that its decisions affect the provision of the city’s public good. A kind of leader-follower equilibrium is obtained, where the school district plays the role of the leader. The results show that the efficiency of provision of the school district public good depends on how the city’s public good is provided. If the city’s public good is underprovided (overprovided) then the school district’s public good is overprovided (underprovided). These outcomes reflect the school district’s attempt to steer the joint choice of public goods toward levels that maximize total property value.
A Appendix

A.1 Determination of the total value of houses

The total value of houses is determined for $t < T_0$ by

$$V(t) = \int_t^{T_0} [R(z_0, s) - (\tau_0 + t_s)V(\tau)] e^{-r(\tau-t)}d\tau + \int_{T_0}^{\infty} [R(z_1, s) - (\tau_0 + t_s)V(\tau)] e^{-r(\tau-t)}d\tau,$$

(A1)

and for $t \geq T_0$

$$V(t) = \int_t^{\infty} [R(z_1, s) - (\tau_0 + t_s)V(\tau)] e^{-r(\tau-t)}d\tau.$$

(A2)

To find $V(t)$ for $t \geq T_0$ (A2) is differentiated with respect to $t$, and the following expression is obtained

$$V_t(t) = - [R(z_1, s) - (\tau_0 + t_s)V(t)] + r \int_t^{\infty} [R(z_1, s) - (\tau_0 + t_s)V(\tau)] e^{-r(\tau-t)}d\tau,$$

(A3)

where $V_t(t)$ is the derivative of $V(t)$ with respect to time.

Taking into account that the last term on the right hand side is equal to $rV(t)$ then (A3) collapses to the following differential equation

$$V_t(t) - (\tau_0 + t_s + r)V(t) = -R(z_1, s).$$

(A4)

To solve (A4), first both sides are multiplied by $e^{-(\tau_0+t_s+r)t}$

$$V_t(t)e^{-(\tau_0+t_s+r)t} - (\tau_0 + t_s + r)V(t)e^{-(\tau_0+t_s+r)t} = -R(z_1, s)e^{-(\tau_0+t_s+r)t}.$$  

(A5)

Integrating both sides between $t$ and $\infty$,\(^{15}\) the following expression is obtained

$$V(t)e^{-(\tau_0+t_s+r)t} = \int_t^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)\tau}d\tau.$$

(A6)

Then $V(t)$ for $t \geq T_0$ is given by

$$V(t) = \int_t^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)(\tau-t)}d\tau.$$  

(A7)

Solving this integral yields

$$V(t) = \frac{R(z_1, s)}{\tau_0 + t_s + r}.$$  

(A8)

\(^{15}\)Notice that the left hand side of (A5) is equal to $\frac{d}{dt}[V(t)e^{-(\tau_0+t_s+r)t}]$. 

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To solve for $t < T_0$ the last term on equation (A1) is replaced by (A7) obtaining
\begin{equation}
V(t) = \int_t^{T_0} [R(z_0, s) - (\tau_0 + t_s)V(t)] e^{-r(\tau-t)} d\tau + e^{-r(T_0-t)} \int_{T_0}^{\infty} R(z_1, s) e^{-(\tau_0+t_s+r)(\tau-T_0)} d\tau. \tag{A9}
\end{equation}
Differentiating (A9) with respect to $t$
\begin{equation}
V_t(t) = -[R(z_0, s) - (\tau_0 + t_s)V(t)] + r \int_t^{T_0} [R(z_0, s) - (\tau_0 + t_s)V(t)] e^{-r(\tau-t)} d\tau
+ r e^{-r(T_0-t)} \int_{T_0}^{\infty} R(z_1, s) e^{-(\tau_0+t_s+r)(\tau-T_0)} d\tau. \tag{A10}
\end{equation}
Again noting that the last part of equation (A10) is equal to $V(t)$, the following differential equation is obtained
\begin{equation}
V_t(t) - (\tau_0 + t_s + r)V(t) = -R(z_0, s). \tag{A11}
\end{equation}
Solving equation (A11) yields
\begin{equation}
V(t)e^{-(\tau_0+t_s+r)t} = \int_t^{T_0} R(z_0, s)e^{-(\tau_0+t_s+r)\tau} d\tau + K, \tag{A12}
\end{equation}
where $K$ is a constant. To find $K$ notice that when $t = T_0$ the total value is
\begin{equation}
V(T_0) = \int_{T_0}^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)(\tau-T_0)} d\tau, \tag{A13}
\end{equation}
and then
\begin{equation}
K = e^{-(\tau_0+t_s+r)T_0} \int_{T_0}^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)(\tau-T_0)} d\tau. \tag{A14}
\end{equation}
Finally putting everything together,
\begin{equation}
V(t)e^{-(\tau_0+t_s+r)t} = \int_t^{T_0} R(z_0, s)e^{-(\tau_0+t_s+r)\tau} d\tau + e^{-(\tau_0+t_s+r)T_0} \int_{T_0}^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)(\tau-T_0)} d\tau, \tag{A15}
\end{equation}
And then,
\begin{equation}
V(t) = \int_t^{T_0} R(z_0, s)e^{-(\tau_0+t_s+r)(\tau-t)} d\tau + \int_{T_0}^{\infty} R(z_1, s)e^{-(\tau_0+t_s+r)(\tau-t)} d\tau. \tag{A16}
\end{equation}
Solving the integrals yields
\begin{equation}
V(t) = \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0+t_s+r)(T_0-t)}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-(\tau_0+t_s+r)(T_0-t)}. \tag{A17}
\end{equation}
In summary, when $0 \leq t \leq T_0$
\begin{equation}
V(t) = \frac{R(z_0, s)}{\tau_0 + t_s + r} (1 - e^{-(\tau_0+t_s+r)(T_0-t)}) + \frac{R(z_1, s)}{\tau_0 + t_s + r} e^{-(\tau_0+t_s+r)(T_0-t)}, \tag{A18}
\end{equation}
and when $t \geq T_0$
\begin{equation}
V(t) = \frac{R(z_1, s)}{\tau_0 + t_s + r}. \tag{A19}
\end{equation}
References


